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DEVELOPMENT OF LONGITUDINAL EQUIVALENT SYSTEM
MODELS FOR SELECTED U.S. NAVY TACTICAL AIRCRAFT

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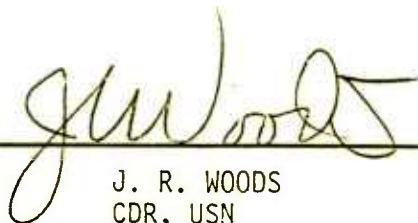
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Frequency response matching techniques were applied to the longitudinal dynamics of five Navy tactical aircraft: the A-6, A-7, S-3, F-14 and F-18, to obtain equivalent system models of their short period response. The pitch rate transfer function was matched with a first order numerator over second order denominator equivalent system model. Freeing the numerator root ($L\alpha$) in the matching process resulted in large unrealistic values of the numerator term. The variation in $L\alpha$ was reduced by simultaneously matching the pitch rate and normal acceleration transfer functions with | | |

the denominators constrained to be equal. The mismatch parameter associated with aircraft configurations which were not amenable to the first over second order equivalent models could be improved by identifying additional roots in the equivalent model. Correlation of the data with the requirements of MIL-F-8785C was straightforward for both short period damping ratio and time delay. Correlation of the frequency requirements, however, required that the control anticipation parameter definition be modified to represent the maximum rather than the initial pitch acceleration response. Defining an attenuated control anticipation parameter allowed for correlation of the high and various low order system representations. All of the aircraft exhibited level 1 flying qualities with the exception of the F-14 response to pilot force inputs which resulted in level 2 time delay and frequency characteristics. Additional effort is required to define the MIL-F-8785C Category C frequency requirements in terms of the attenuated control anticipation parameter.

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LIST OF SYMBOLS

Symbols

| | | |
|----------------|---|----------------------------------|
| CAP | — | Control Anticipation Parameter |
| CAP' | — | Attenuated CAP |
| F | — | Stick force — lb |
| g | — | Gravitational acceleration |
| K | — | System gain |
| L_{α} | — | Lift curve slope — ft/sec |
| M | — | Mismatch parameter |
| n_z | — | Normal acceleration — g's |
| s | — | Laplace operator |
| t | — | Time — sec |
| $T_{\theta 2}$ | — | Pitch rate num. time cons. — sec |
| V | — | True airspeed — ft/sec |
| α | — | Angle of attack — rad |
| θ | — | Pitch angle — rad |
| δ | — | Cockpit control deflection — rad |
| δ_e | — | Control surface deflection — rad |
| ζ | — | Damping ratio |
| ω | — | Frequency — rad/sec |
| τ | — | Time delay — sec |
| . | — | Time derivative |

Subscripts

| | | |
|-----|---|-------------------|
| e | — | Equivalent System |
| HOS | — | High Order System |
| LOS | — | Low Order System |
| nd | — | Non-dimensional |
| p | — | Pilot input |
| sp | — | Short period |
| ss | — | Steady state |

Transfer Function Root Representation

| | | |
|-------|---|--|
| (a) | — | Real root of value a |
| [b,c] | — | Imaginary root with damping b, frequency c |

INTRODUCTION

BACKGROUND

The Military Specification for Flying Qualities of Piloted Airplanes, MIL-F-8785B, (reference (a)), was developed largely from flight tests of classically responding unaugmented aircraft. Its quantitative requirements are generally expressed in terms of modal approximations which can be described mathematically by first or second order linear expressions. Advancements in aerodynamics and complicated control system augmentation schemes, prevalent in modern aircraft designs, have resulted in responses which are described by high order functions. The high order modal parameters may differ significantly from their linear components.

In an attempt to utilize the existing requirements in analyzing advanced aircraft/control system configurations, the concept of equivalent systems, has been introduced (reference (b)). A digital frequency domain equivalent system matching technique has been developed by Hodgkinson, et. al., and applied to the high order representations of experimental aircraft (references (c) and (d)). The approach used was to approximate the high order pitch rate to pilot control input transfer function of the subject aircraft with a classical low order transfer function describing the specification requirements, augmented with a time delay. This equivalent time delay approximates the phase lag introduced by the high frequency control system components. Within the scope of the initial investigations, it was determined that the linear modal requirements of MIL-F-8785B, when augmented by a requirement on time delay, are appropriate for specifying the handling qualities of the advanced high order configurations of tomorrow's airplanes (reference (e)). This approach has been incorporated in the latest revision to the MIL-SPEC, MIL-F-8785C, reference (f), which states:

"The contractor shall define equivalent classical systems which have responses most closely matching those of the actual aircraft."

The parameters defining the resulting equivalent system (frequency, damping ratio, etc.) rather than any modes of the high order system, are to be compared with the specification requirements. However, no guidance is given as to how the contractor shall proceed with his equivalent system definition nor with what criteria its adequacy will be judged by the procuring agency.

The Naval Air Development Center, as part of its effort in identifying flying qualities criteria for manned aircraft, undertook the determination of equivalent system descriptions of current Navy tactical aircraft. This investigation not only encompassed the determination of the classical pitch-rate short-period model for current fleet aircraft, but also investigated the effects of various high order configuration components and additional aircraft response parameters.

PURPOSE

The purpose of this effort was to investigate the utility of the equivalent systems approach to defining the dynamic longitudinal flying qualities parameters of augmented aircraft. This report presents equivalent low order system models for current U.S. Navy tactical aircraft and compares them with the modal requirements of MIL-F-8785C, reference (f).

SCOPE

The low order equivalent systems presented in this report were determined via frequency response matching techniques. The relative merits of frequency versus time response matching

have been addressed elsewhere (reference (g)), and will not be further discussed in this report. The frequency response matching technique was chosen for this effort due to its more definitive state of development and the fact that it is currently being investigated throughout the aircraft industry.

The aircraft included in this analysis were the A-6, A-7, F-14, F-18 and S-3. Where applicable, each aircraft was assumed to have its Stability/Control Augmentation System (SAS/CAS) ON. Only longitudinal dynamics were analyzed. The flight conditions investigated included both Power Approach (PA) and Cruise (CR) configurations as presented in table I.

TABLE I. FLIGHT CONDITIONS

| Aircraft | Configuration | Gross Weight (lb) | CG Position (% MAC) | Altitude (ft) | Airspeed (M/KEAS) |
|----------|---------------|-------------------|---------------------|---------------|---------------------------|
| A-6 | CR | 35905 | 23.6 | 20000 | 0.4 0.72 0.88 95 |
| | PA | 26600 | 24.1 | 0 | |
| S-3 | CR | 36320 | 21.7 | 15000 | 0.36 0.71 97 |
| | PA | 31790 | 25.0 | 0 | |
| F-14 | CR | 51015 | 8.2 | 15000 | 0.5 |
| | | | 8.2 | | 0.7 |
| | | | 10.4 | | 0.83 |
| | | | 11.4 | | 1.2 |
| | PA(1)(1) | 46380 | 9.8 | 0 | 126 |
| | PA(2)(2) | | 9.8 | 0 | 121 |
| A-7 | CR | 21890 | 30.0 | 15000 | 0.3 |
| | | | | | 0.6 |
| | | | | | 0.9 |
| F-18 | CR | 29930 | 25.0 | 10000 | 0.5 |
| | PA | 30700 | 25.0 | 0 | 133 |

Notes: (1) Direct Lift Control (DLC) ON
(2) DLC OFF

METHOD

Frequency response matching techniques were utilized to determine low order equivalent systems describing the complex aircraft high order representations. Digital computer programs, prepared by the McDonnell Aircraft Company, utilized a direct Rosenbrock search algorithm (reference c) to match a Bode plot describing the high order pilot input to aircraft output transfer function with an equivalent low order system. Since this analysis is concerned with determining equivalent longitudinal short period models, the pitch rate and normal acceleration responses to pilot control inputs were analyzed.

In order to use the matching routines, a description of the frequency response of the system to be matched is required. This may be in the form of either (1) transfer functions or (2) numerical phase-gain data obtained at various input frequencies. Since only limited numerical response data is available for the subject airplanes (and it is generally corrupted with instrumentation noise and air turbulence) the transfer function input approach was chosen. Each aircraft's transfer functions describing the desired responses were obtained either directly from available information (A-7) or computed via NADC transfer function programs from stability and control derivative information (A-6, S-3, F-14, F-18). References (h) through (m) were used to obtain this information as well as a description of the respective control systems. With the aircraft's unaugmented dynamics thus obtained, the control components present in each aircraft's control system (i.e., actuators, stick feed system, feedback loops, compensation networks, etc.) were added to obtain the high order transfer function describing each aircraft/control system combination and flight condition. Brief descriptions of the aircraft and their respective control systems are presented in appendix A.

Two computer programs, LONFIT (reference n) and NAVFIT (reference o) were utilized to obtain the frequency response matches presented in this report. The LONFIT program utilized a low order system of the form:

$$\frac{\dot{\theta}}{\delta_x} = \frac{K_{\theta} (s + L_{\alpha}) e^{-\tau_{\theta}s}}{s^2 + 2\zeta_{sp} \omega_{sp} s + \omega_{sp}^2} \quad (1)$$

when matching the pitch rate response, and

$$\frac{n_z}{\delta_x} = \frac{K_{nz} e^{-\tau_{nz}s}}{s^2 + 2\zeta_{sp} \omega_{sp} s + \omega_{sp}^2} \quad (2)$$

when matching the normal acceleration response. The NAVFIT program is a general program which allows the analyst to choose the order of the assumed low order system.

The match was obtained by minimizing the sum of the squares of the difference in both magnitude and phase of the high and low order system as expressed by:

$$M = \sum \left\{ (\text{Gain}_{\text{HOS}} - \text{Gain}_{\text{LOS}})^2 + 0.01745 (\text{Phase}_{\text{HOS}} - \text{Phase}_{\text{LOS}})^2 \right\} \quad (3)$$

where gain is in decibels and phase in degrees.

The summation was performed at a number of discrete intervals (typically 21) over the pilot's short period frequency range of interest (nominally .1 to 10 rad/sec).

In performing the match, the analyst worked interactively with the program to determine which of the decision variables (K_{θ} , L_{α} , ζ_{sp} , ω_{sp} , τ_{θ}) to vary in the search for the equivalent low order system. In general, the McAIR recommended procedure, outlined below, was utilized to introduce decision variables into the search.

- a. Initially, attempt to match the high order system response in the frequency range of 0.1 to 10 rad/sec by varying gain or zeta or omega or all three, with L_{α} fixed at the estimated airplane value, and with no time delay.
- b. If, after obtaining a plot of the high order system gain characteristics, it is evident that the 0.1 through 10 rad/sec frequency range does not include the peak in gain, expand

the frequency range to include the peak. If two separate peaks are evident in the 0.1 through 10 rad/sec frequency range, modify the range to span only the assumed short period peak.

- c. If a good match (a payoff function less than 20 is considered a good match) cannot be obtained after following steps a and b, include time delay in the search. (Inclusion of the time delay is usually effective when a mismatch in phase at higher frequencies is evident.)
- d. If after following steps a, b and c, a low payoff has not resulted, delete the time delay and include L_{α} in the search. (Inclusion of L_{α} is usually effective when a mismatch in low and intermediate frequencies is evident.)
- e. If after following steps a, b, c, and d, a good match has not been obtained, include both L_{α} and time delay in the search.
- f. If after the above procedure, an acceptable payoff function cannot be obtained, the configuration dynamics must be considered to not be represented by the assumed low order system and an alternative evaluation method is necessary.

In performing these matches, the "goodness of fit" was determined quantitatively by the value of the mismatch parameter and qualitatively from Bode plots and time history responses to unit impulse and step control inputs. Frequency and time history comparisons for representative equivalent systems examples are presented in appendix B.

The resulting modal parameters representing the low order systems, as well as the dominant roots of the high order system, were compared against the requirements of MIL-F-8785C. The dominant roots of the high order system were defined as that oscillatory pair present in the frequency range from 0.5 to 10 rad/sec.

RESULTS AND DISCUSSION

GENERAL

Equivalent system models were initially obtained for the pitch rate to cockpit control position transfer functions for the A-6, S-3 and F-14 airplanes. In these cases, the matching procedure was straightforward, resulting in excellent frequency and time history response matches. Subsequently, control stick dynamics were included in the F-14 airplane's high order system model and equivalent pitch rate to control force transfer functions were generated. Control force was also used as the input command for the A-7 and F-18 airplanes, although in these cases, control stick dynamics were not modeled.

The inclusion of feel system dynamics compounded the matching problem by introducing additional roots within the short period frequency range of interest. As a result, these equivalent systems resulted in either large mismatch parameters or large numerator root (L_{α}) values which are inconsistent with the airplane's lift curve slope.

Two methods of restricting the variation in L_{α} were analyzed. First, the normal acceleration response was included into the classical short period approximation matching procedure. Matching the high order normal acceleration at the instantaneous center of rotation provided a short period

characteristic equation consistent with that obtained from the L_α fixed pitch rate analysis. In addition, simultaneously matching pitch rate and normal acceleration significantly reduced the variation in equivalent L_α . Secondly, since the majority of the mismatch evident with L_α fixed occurred in the vicinity of the additional feel system roots, the equivalent short period definition was modified to account for this additional root. Matching pitch rate and normal acceleration with first over third and zero over third order transfer functions, respectively, resulted in low mismatch values and excellent time history comparisons.

Frequency and time history comparisons for each of the identified equivalent systems models are presented in appendix B.

Comparison of the equivalent time delay and damping ratio with the requirements of MIL-F-8785C were straightforward. This may be attributed to the fact that the requirements are imposed singularly on the parameters of interest and are not correlated with any other parameters. The short period frequency requirement, however, correlates frequency (ω_{sp}) with acceleration sensitivity (n/α) via boundaries of constant control anticipation parameter ($\omega_{sp}^2/n/\alpha$). It is a simple matter to plot equivalent short period frequency as a function of equivalent acceleration sensitivity. However, the resulting classical equivalent short period control anticipation parameter does not correlate with the high order system's control anticipation parameter ($\dot{\theta}_{max}/n_{z_{ss}}$). To alleviate this problem a control anticipation parameter was developed which accounts for the attenuation affects of the high order control system (reference (p)). The use of this parameter results in a consistent description of both the high and low order systems which can be correlated to pilot opinion boundaries.

CLASSICAL SHORT PERIOD EQUIVALENT SYSTEMS

The short period mode of motion of interest is primarily characterized by a rapid rotation in aircraft attitude. Therefore, the major emphasis in longitudinal equivalent systems matching has been centered on the pitch response, the parameters of which are necessary for comparison with the requirements of MIL-F-8785C. The same approach has been taken in this analysis.

Initial matches were obtained for the pitch rate to pilot control position input transfer functions for the A-6, S-3 and F-14 airplanes. In general, these configurations were characterized by the bare airframe augmented by control surface actuators and pitch rate feedbacks. From these configurations, a basic understanding of the frequency response matching process was obtained. Subsequently, the effect of compounding dynamic components such as control stick dynamics, normal acceleration feedbacks and system prefilters were investigated by matching pitch rate to pilot force inputs for the F-14, A-7 and F-18 aircraft.

A-6 AIRPLANE

The A-6 airplane utilizes washed out pitch rate, as described in appendix A, to augment the airplane's basic stability characteristics. The resulting pitch rate response to pilot longitudinal control position inputs can be represented by a fourth order numerator over sixth order denominator transfer function. The pitch rate feedback component adds a numerator and denominator root in the vicinity of 0.5 rad/sec while the actuator adds a single denominator root at approximately 30 rad/sec. The proximity of the added feedback roots results in their effectively cancelling one another while the actuator root is quite far removed from the closed loop short period root. It is to be expected therefore, that the equivalent roots identified by the frequency matching procedure will quite closely match the "dominant roots" of the high order system augmented with a time delay.

The McAir recommended matching procedure was utilized in the NAVFIT program to determine a first order numerator over second order denominator equivalent system for each of the flight conditions analyzed. The complete procedure was followed, even though acceptable matches were obtained by varying only gain, damping ratio and frequency, in order to obtain an understanding of the effects of introducing different decision variables into the search. The A-6 high and low order equivalent systems are summarized in table II.

With the phugoid roots included in the high order model, the frequency range over which the match was conducted had to be modified to exclude their contributions. The effect of varying the match frequency range to exclude phugoid contributions is presented in figure 1. With the frequency matching range including the tail of the phugoid resonant peak, the equivalent short period damping and frequency are different from that of the high order system dominant root pair and a high mismatch results.

Modifying the frequency range to exclude the phugoid contributions yields results which correspond with those obtained from removing the phugoid roots from the high order system prior to performing the match. In the cases in which the phugoid contributions were not excluded, the search routine is attempting to match the gain and phase characteristics of two resonant peaks with only one set of numerator and denominator breakpoints. As a result, a compromise is achieved which provides the best average mismatch across the entire frequency range, but only approximates the desired short period characteristics.

In the majority of the cruise configurations analyzed, the phugoid and short period roots were widely enough separated that modifying the match frequency range produced acceptable results. However, in the power approach cases, the phugoid and short period roots were much more closely coupled. In these instances, increasing the low frequency boundary to exclude phugoid effects also cut off some of the lower frequency short period information. For the A-6 power approach case, the lower frequency bound was raised to 0.5 rad/sec to exclude phugoid contributions. However, the aircraft's numerator term, $1/T_{\theta 2}$, is at 0.461 rad/sec. Therefore, using 0.5 rad/sec as the lower frequency bound cuts off a portion of the short period, as well as the phugoid response. Although acceptable matches were attainable by using a lower frequency bound of 0.5 rad/sec, the resulting short period parameters will later be shown to have less than satisfactory correlation with the high order modal parameters. In instances such as this, where the phugoid and short period roots were closely coupled, the phugoid roots were removed from the high order model prior to performing the equivalent system match.

With the frequency range modified to exclude the phugoid contributions to the gain and phase plots, the major improvement in system match was obtained via the time delay parameter. The time delay serves to modify the phase characteristics of the low order system to account for the phase characteristics attributable to the high frequency terms in the high order system. It has unity gain and therefore affects only the Bode phase characteristics. Inclusion of the time delay parameter had little effect on the identified frequency and damping ratios but greatly reduced the mismatch parameter by improving the phase match at the higher frequencies, as shown in figure 2. The equivalent pitch rate time history is also improved, although there is no response from $t = 0$ to $t = \tau$ and the initial value of pitch acceleration is much higher than that of the high order system in order to match the slope of the pitch rate response following the delay.

Freeing L_{α} in the search process also improved the quality of the match, although generally not as much as the time delay parameter did. As can be seen from figure 3, freeing L_{α} primarily improved the match at frequencies below the short period frequency and only slightly improved

TABLE II. A-6 AIRPLANE EQUIVALENT TRANSFER FUNCTIONS
Pitch Rate Response to Cockpit Control Position Inputs

| Config- uration | Altitude (ft) | Airspeed (M/KEAS) | Match Variables | | | Equivalent System Parameters | | | | | | | | | |
|--------------------|------------------|----------------------|-----------------------|------------|----------|---------------------------------|-------------------|------------|--------------|---------------|--------|----------|------|-------|------|
| | | | K, ζ , ω | L_α | τ | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch | | | |
| CR | 20,000 | 0.4/179 | High Order System | | | 4.31 | 0.506 | 0.63 | 2.32 | — | — | — | | | |
| | | | X | | X | 0.1 — 10 | 0.155 | 0.506 | 0.83 | 2.05 | 0.027 | 646.0 | | | |
| | | | X | | X | 0.3 — 10 | 0.126 | 0.506 | 0.59 | 2.19 | — | 13.6 | | | |
| | | | X | | | ↓ | 0.134 | 0.506 | 0.64 | 2.27 | 0.029 | 1.8 | | | |
| | | | X | X | 0.122 | | 0.635 | 0.54 | 2.32 | — | 10.2 | | | | |
| | | | X | X | 0.132 | | 0.564 | 0.61 | 2.31 | 0.027 | 1.2 | | | | |
| | | | X | | 0.132 | | 0.506 | 0.62 | 2.3 | 0.029 | 0.1 | | | | |
| CR | 20,000 | 0.72/323 | High Order System | | | 13.94 | 1.077 | 0.86 | 4.86 | — | — | — | | | |
| | | | X | | | 0.3 — 10 | 0.397 | 1.077 | 0.80 | 4.2 | — | 11.6 | | | |
| | | | X | | X | ↓ | 0.507 | 1.077 | 0.93 | 4.75 | 0.036 | 2.8 | | | |
| | | | X | X | 0.363 | | 1.527 | 0.66 | 4.60 | — | 4.7 | | | | |
| | | | X | X | 0.444 | | 1.344 | 0.78 | 4.84 | 0.027 | 0.5 | | | | |
| | | | CR | 20,000 | 0.88/395 | | High Order System | | | 19.99 | 1.341 | 0.93 | 6.82 | — | — |
| | | | | | | X | | | 0.3 — 10 | 0.558 | 1.341 | 0.85 | 5.49 | — | 10.6 |
| X | | X | | | | ↓ | 0.865 | 1.341 | 1.05 | 6.84 | 0.040 | 3.6 | | | |
| X | X | 0.494 | | | | | 1.89 | 0.68 | 5.94 | — | 3.9 | | | | |
| X | X | 0.655 | | | | | 1.682 | 0.82 | 6.51 | 0.027 | 1.2 | | | | |
| PA | Sea Level | 0.14/95 | | | | | High Order System | | | 4.26 | 0.461 | 0.71 | 1.46 | — | — |
| | | | | | | X | | X | 0.5 — 10 | 0.134 | 0.461 | 0.81 | 1.27 | 0.032 | 5.5 |
| | | | X | X | ↓ | 0.129 | 0.854 | 0.7 | 1.54 | 0.025 | 2.4 | | | | |
| | | | X | | | 0.129 | 0.578 | 0.71 | 1.44 | 0.030 | 0.2 | | | | |

Note: (1) Phugoid contributions removed from high order system

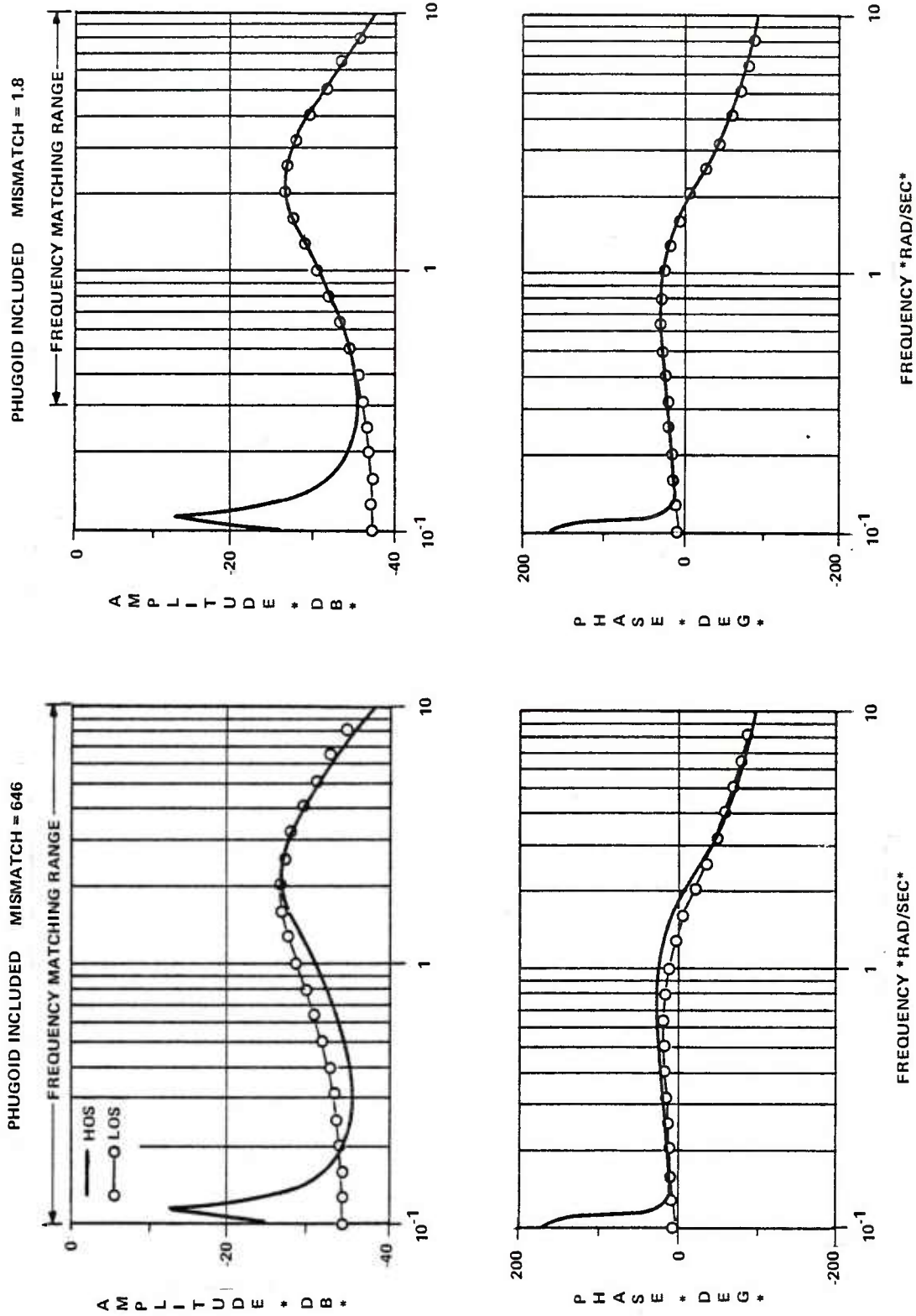


Figure 1 — Phugoid Effects on Low Order System Determination — A-6 Airplane — .4M/20,000 ft Altitude

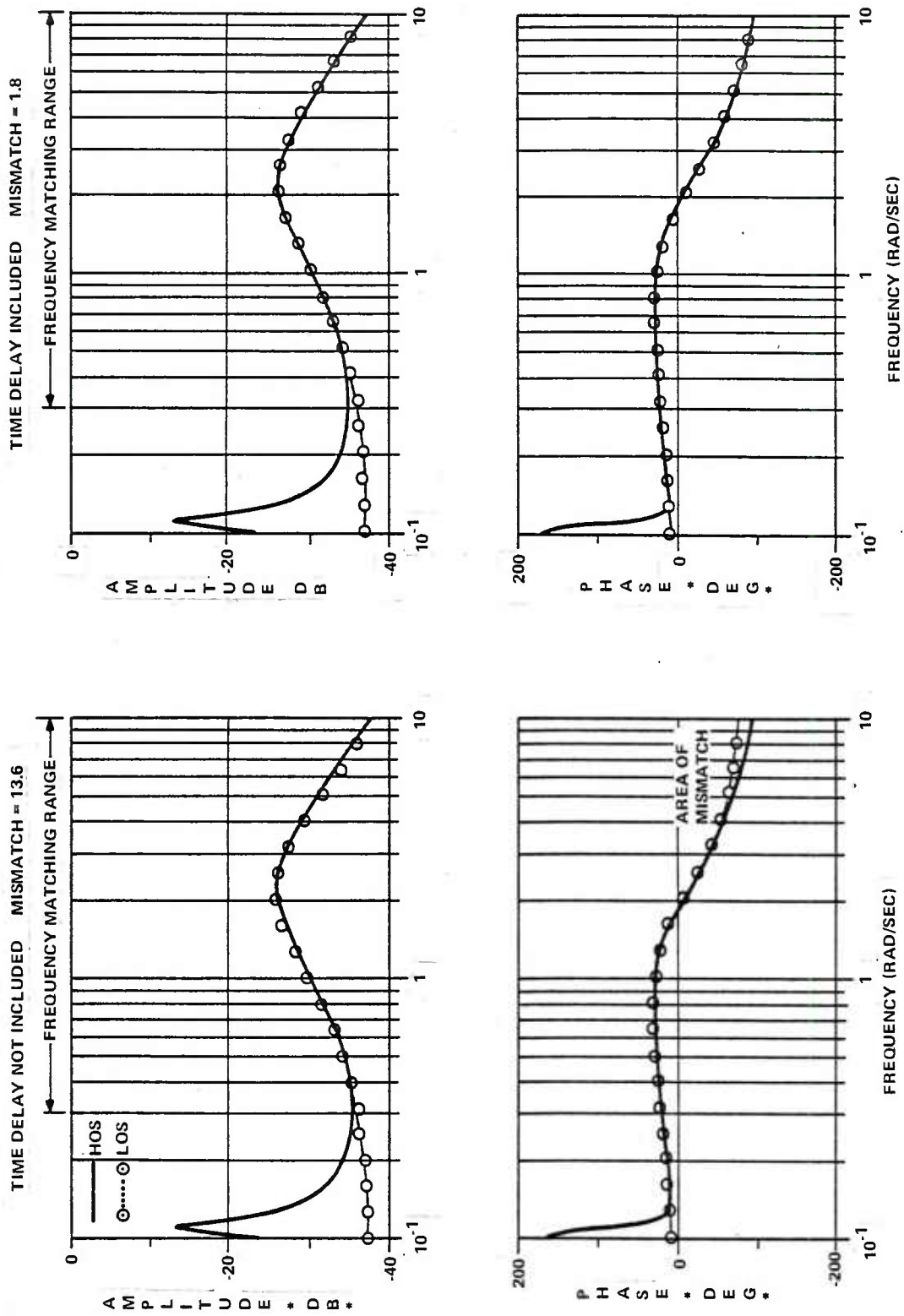


Figure 2 -- Time Delay Effects -- A-6 Airplane -- 4M/20,000 ft. Altitude

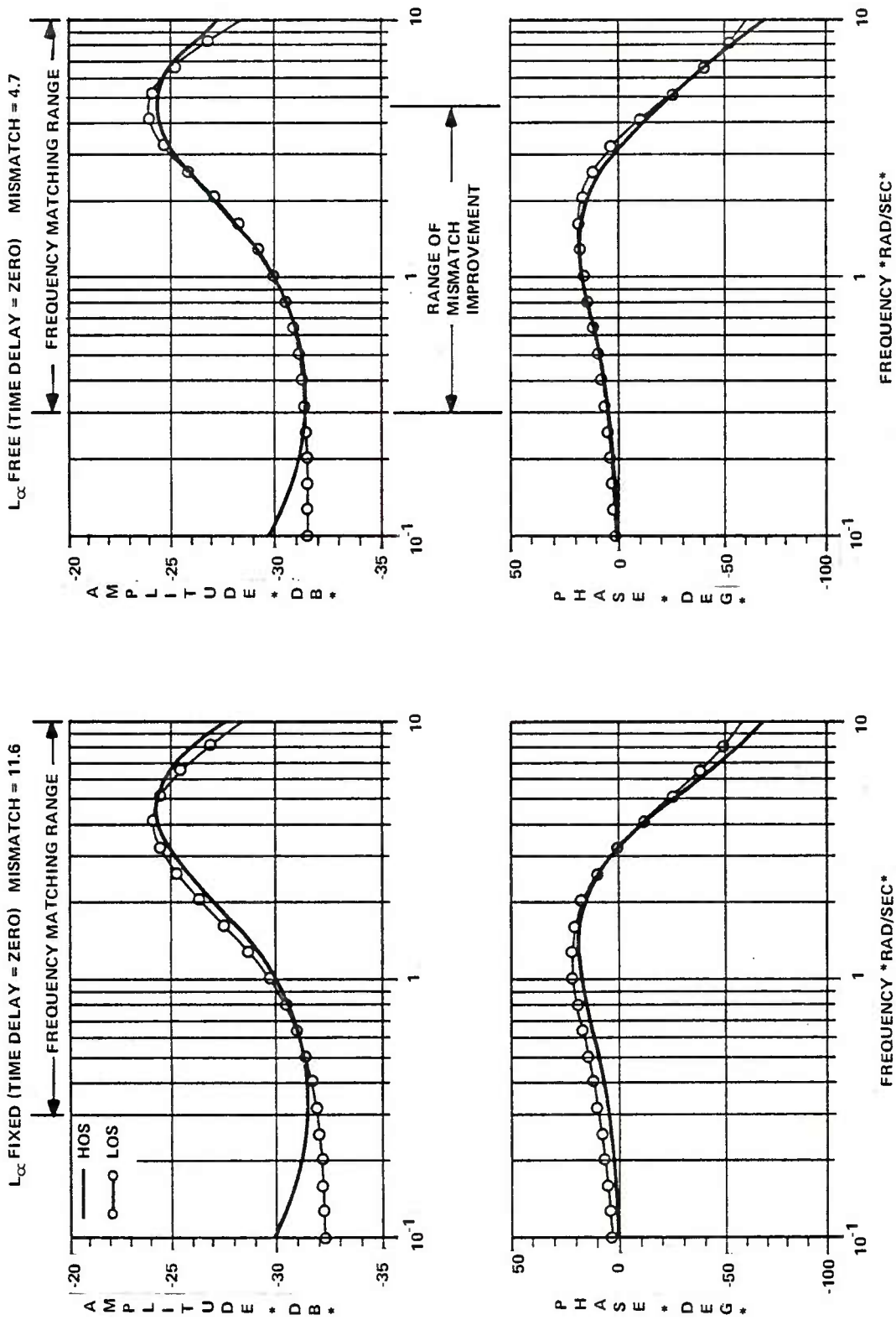


Figure 3 — Effect of Freeing L_α — A-6 Airplane — .72M/20,000 ft. Altitude

the match at the higher frequencies. Freeing $L\alpha$, with zero time delay, resulted in higher values of $L\alpha$ than freeing both $L\alpha$ and time delay due to the attempt at matching the high frequency phase characteristics with fewer parameters.

Based on these results, phugoid contributions were excluded from the matching process, time delay was included in all subsequent matches, and $L\alpha$ was allowed to vary, where necessary, to improve the mismatch parameter.

S-3 AIRPLANE

There are no stability augmentation components included in the longitudinal control system of the S-3 airplane. Its high order representation consists of unaugmented phugoid and short period dynamics controlled by an elevator actuator with a 0.0294 sec. time constant.

The NAVFIT frequency matching routine readily identifies the S-3's short period dynamics with equivalent delays of 0.027–0.029 sec. as presented in table III. The power approach characteristics again necessitate the removal of phugoid dynamics from the high order representation, due to the close coupling between short period and phugoid roots.

F-14 AIRPLANE

Pitch Rate Response Matching — The F-14 airplane's control system includes a washed out pitch rate feedback signal that is passed through a shaping network to provide the desired response to command inputs. The resulting F-14 high order pitch rate to pilot control position transfer function can be represented by a sixth order numerator over eighth order denominator.

With L_v fixed at the value obtained from the F-14 airplane's aerodynamic characteristics, acceptable low order equivalent systems (mismatch ≈ 10) are readily obtained, as presented in table IV. This may be attributed to the observation that although there are a large number of roots present in the frequency range of interest, each closed loop numerator root is accompanied by a similar denominator root and therefore only effects the response over a limited range of frequencies.

It can also be seen that freeing $L\alpha$ results in a significant improvement in the mismatch parameter to values less than 1. This reduction in mismatch is accompanied by increases in $L\alpha$ and reductions in time delay, damping ratio and gain. The frequency increases for subsonic flight conditions and decreases for the one supersonic case.

The feel-system in the F-14 airplane includes a sprashpot to damp out control stick oscillations and a feel spring to provide pilot force feel of commanded inputs. This feel system may be represented by a first over third order transfer function as shown in appendix A. The addition of this feel system to the augmented F-14 airplane's pitch dynamics results in a seventh order numerator over eleventh order denominator pitch rate transfer function.

The equivalent systems obtained for the stick force command inputs are included in table IV. Matching the stick force command high order transfer functions with first over second order equivalent systems resulted in significantly different modal parameters from those obtained in the control position analysis. With L_v fixed, the equivalent frequency is lower and the time delay and mismatch parameters are considerably higher.

TABLE III
S-3 AIRPLANE PITCH RATE EQUIVALENT TRANSFER FUNCTIONS

| Configuration | Altitude (ft) | Airspeed (M/KEAS) | Match Variables | | Equivalent System Parameters | | | | | |
|---------------|------------------|----------------------|-----------------|---------------------------------|------------------------------|------------|--------------|---------------|--------|----------|
| | | | L_α | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch |
| CR | 15,000 | 0.36/179 | | HOS | 249.2 | 0.714 | 0.44 | 2.59 | — | — |
| | | | Fixed | 0.3—10 | 7.365 | 0.714 | 0.45 | 2.55 | 0.029 | 1.8 |
| CR | 15,000 | 0.71/353 | | HOS | 786.7 | 1.766 | 0.48 | 5.45 | — | — |
| | | | Fixed | 0.1—10 | 22.69 | 1.766 | 0.48 | 5.39 | 0.027 | 0.3 |
| PA | Sea Level | 0.15/97 | | HOS | 73.43 | 0.594 | 0.57 | 1.53 | — | — |
| | | | Fixed | 0.5—10 | 2.19 | 0.594 | 0.61 | 1.43 | 0.031 | 3.1 |
| | | | Fixed | 0.1—10 ⁽¹⁾ | 2.13 | 0.675 | 0.57 | 1.51 | 0.028 | 0.1 |

Note: (1) Phugoid contributions removed from high order system

TABLE IV
F-14 AIRPLANE PITCH RATE EQUIVALENT TRANSFER FUNCTIONS

| Configuration | Altitude | Airspeed (M/KEAS) | Transfer Function | Match Variables | | Equivalent System Parameters | | | | | |
|-------------------|--------------|----------------------|--|-----------------------|---------------------------------|------------------------------|------------|--------------|---------------|--------|----------|
| | | | | L_α | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch |
| CR | 15,000 | 0.5/248 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. | | — | 0.773 | 0.61 | 2.78 | — | — |
| | | | | Fixed | 0.3–10 | .277 | 0.773 | 0.76 | 2.36 | 0.052 | 10.9 |
| | | | | Free | ↓ | .245 | 1.334 | 0.58 | 2.74 | 0.039 | 0.6 |
| | | | | Fixed | | .0278 | 0.773 | 0.64 | 1.74 | 0.171 | 71.3 |
| CR | 15,000 | 0.7/348 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. | | — | 0.962 | 0.58 | 4.27 | — | — |
| | | | | Fixed | 0.3–10 | .623 | 0.962 | 0.75 | 4.08 | 0.050 | 11.4 |
| | | | | Free | ↓ | .516 | 1.443 | 0.56 | 4.31 | 0.034 | 0.8 |
| | | | | Fixed | | .0548 | 0.962 | 0.74 | 2.75 | 0.153 | 37.3 |
| CR | 15,000 | 0.83/412 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. | | — | 0.965 | 0.62 | 5.01 | — | — |
| | | | | Fixed | 0.3–10 | .835 | 0.965 | 0.79 | 5.01 | 0.048 | 8.7 |
| | | | | Free | ↓ | .675 | 1.347 | 0.61 | 5.07 | 0.032 | 0.4 |
| | | | | Fixed | | .067 | 0.965 | 0.82 | 3.19 | 0.142 | 22.7 |
| CR | 15,000 | 1.2/596 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. | | — | 1.143 | 0.71 | 6.88 | — | — |
| | | | | Fixed | 0.3–10 | 1.911 | 1.143 | 0.94 | 8.64 | 0.045 | 6.8 |
| | | | | Free | ↓ | 1.198 | 1.485 | 0.67 | 7.50 | 0.021 | 0.9 |
| | | | | Fixed | | .108 | 1.143 | 1.07 | 4.56 | 0.114 | 7.9 |
| PA ⁽¹⁾ | Sea Level | 0.19/126 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. ⁽¹⁾ | | — | 1.919 | 0.73 | 4.94 | 0.095 | 0.9 |
| | | | | Fixed | 0.1–10 | .059 | 0.473 | 0.52 | 1.48 | — | — |
| | | | | Fixed | ↓ | .0071 | 0.473 | 0.51 | 1.38 | 0.049 | 2.1 |
| | | | | Free | | .006 | 1.214 | 0.41 | 1.14 | 0.188 | 102.7 |
| PA ⁽²⁾ | Sea Level | 0.18/121 | $\dot{\theta}/\delta_{ep}$ $\dot{\theta}/F$ | H.O.S. ⁽¹⁾ | | — | 0.444 | 0.7 | 1.06 | — | — |
| | | | | Fixed | 0.1–10 | .052 | 0.444 | 0.64 | 0.97 | 0.049 | 2.0 |
| | | | | Fixed | ↓ | .0065 | 0.444 | 0.49 | 0.81 | 0.19 | 108.5 |
| | | | | Free | | .0041 | 3.456 | 0.44 | 1.54 | 0.143 | 46.2 |

Note: (1) Phugoid components removed from high order system prior to performing match

The pitch response resulting from step control and force inputs are compared in figure 4, where the magnitude of the inputs has been normalized to yield the same steady state pitch rate. The addition of the control stick feel system results in a more sluggish initial pitch response than that obtained for control position inputs (i.e.—the frequency is lower and the response is delayed—as indicated by comparing the force and position equivalent system parameters).

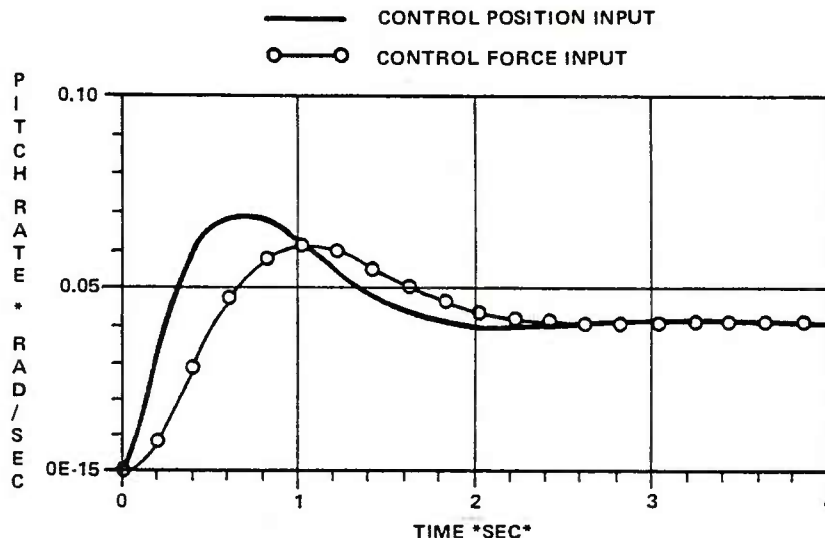
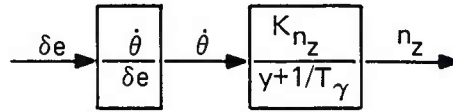


Figure 4 — F-14 Airplane Response to Control Force and Position Commands — .5M/15,000 ft Altitude

Freeing L_{α} reduces the mismatch parameter to values similar to those obtained in the control position analysis. This is accomplished at the expense of L_{α} ; however, which is now seen to "gallop" to extremely large values in an attempt to match the gain characteristics of the high order system. Where freeing L_{α} in the control position cases resulted in increases in L_{α} of 30 to 75 percent, it now increases by 75 to 500 percent. The resulting frequency and time history responses show very good agreement between the high and low order systems (Appendix B, figure B-6). However it is not conceivable that the control system implementation could alter L_{α} to the extent that the equivalent system analysis would indicate.

Normal Acceleration Response Matching — The equivalent systems obtained from the pitch rate transfer function often resulted in conditions of galloping L_{α} . This was most evident for those cases in which the control system introduced additional roots in the pilot's frequency range of interest. The LONFIT program includes the capability of matching aircraft normal acceleration via the equivalent system presented in equation (2).

The normal acceleration thus defined may be viewed as a measure of the aircraft's path response represented as:



where the path response lag parameter, $1/T_\gamma$, may be shown to be approximately equal to the numerator term in the $\theta/\delta e$ transfer function. Therefore matching the $n_z/\delta e$ transfer function should provide information concerning the short period characteristic equation without having to be concerned about matching the numerator roots.

Equation (2) is, however, only valid for normal accelerations measured at the airplane's center of rotation. At that location, the initial vertical acceleration of the tail produced by a step elevator input is balanced by that due to the aircraft's pitching acceleration. At any other location, this condition does not hold and two nonminimum phase numerator roots are introduced in the normal acceleration transfer function:

$$\frac{n_z(s)}{\delta e(s)} = \frac{K_{n_z} (S + 1/T_{n_{z1}}) (S + 1/T_{n_{z2}})}{s^2 + 2\zeta_{sp} \omega_{sp} s + \omega_{sp}^2} \quad (4)$$

Matching normal acceleration at the center of gravity, or the pilot's location, with a zero over second (0/2nd) order transfer function will only approximate the high order system representation. For example consider the F-14 airplane response to control position inputs at 0.5M, 15,000 ft. presented in figure 5.

The zero over second order equivalent system only approximates the high order system normal acceleration at the center of gravity. Although the mismatch parameter is reasonable, the short period frequency, damping ratio and equivalent time delay are significantly different from those previously obtained from the pitch rate transfer function. In addition, the equivalent time response does not match the initial reversal in normal acceleration arising from the nonminimum phase numerator roots. The normal acceleration at the center of rotation is, however, matched at least as well as the pitch rate expression and results in identical frequency and damping ratio and similar time delays.

Simultaneously matching pitch rate and normal acceleration at the center of rotation, with the denominators constrained to be identical, results in a slight improvement in the pitch rate match at the expense of the normal acceleration match. This technique was applied to the F-14 cases in which L_α increased by approximately 100 percent or more when freed in the search routine. The high order representations and resulting equivalent systems are presented in table V.

The equivalent systems obtained for the normal acceleration response at the center of rotation are, in all cases, consistent with those obtained for the pitch rate response with L_α fixed. Simultaneously matching the pitch rate, with L_α free, and normal acceleration transfer functions has the effect of restricting the variation in L_α evident when matching pitch rate alone.

| PARAMETER | ORDER | K_θ | L_α | τ_θ | ζ_{sp} | ω_{sp} | K_{nz} | τ_{nz} | MISMATCH |
|---------------------------------------|-------|------------|------------|---------------|--------------|---------------|----------|-------------|----------|
| | HOS | — | 0.773 | — | 0.61 | 2.76 | — | — | — |
| $\dot{\theta}$ | 1/2 | 0.277 | 0.773 | 0.052 | 0.76 | 2.36 | — | — | 10.9 |
| $N_{z_{cg}}$ | 0/2 | — | — | — | 1.16 | 3.7 | -9.71 | 0.127 | 33.8 |
| $N_{z_{cr}}$ | 0/2 | — | — | — | 0.76 | 2.37 | 3.55 | 0.032 | 9.3 |
| $\dot{\theta}$ and $N_{z_{cr}}$ | 1/2 | 0.268 | 0.885 | 0.048 | 0.73 | 2.41 | | | 7.7 |
| | 0/2 | | | | 0.73 | 2.41 | 3.57 | 0.033 | 10.7 |

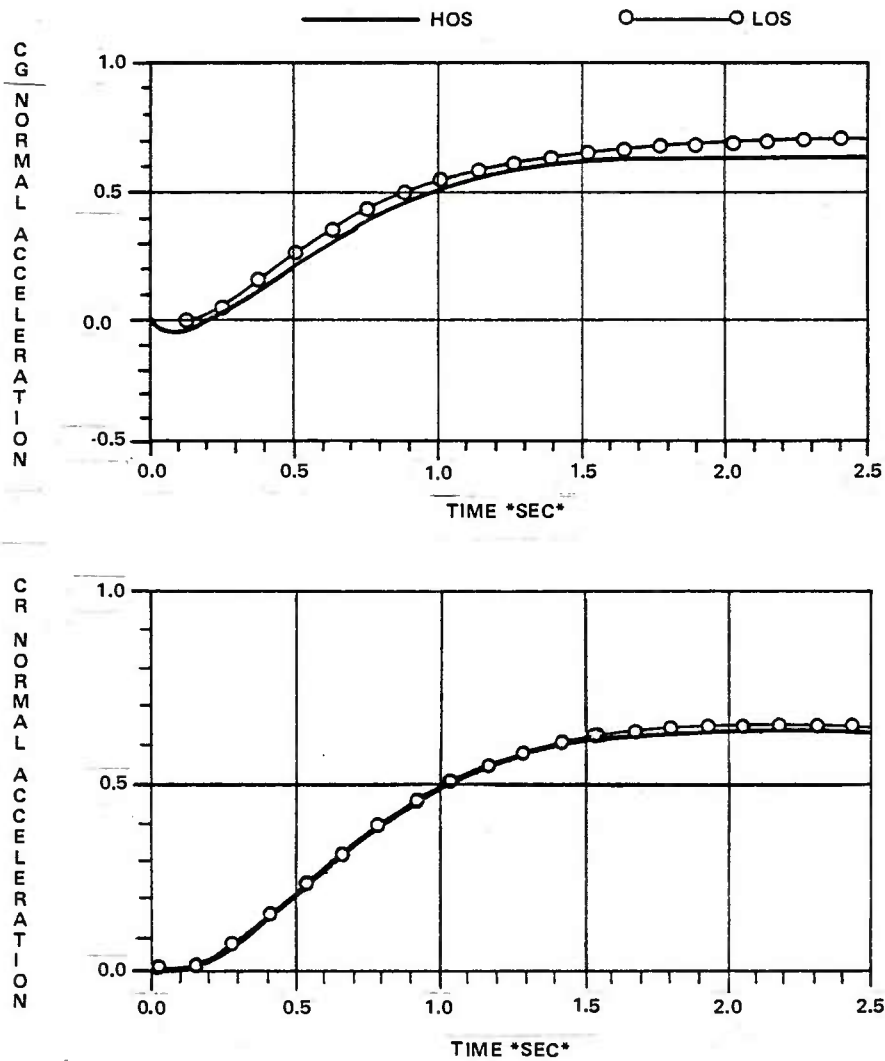


Figure 5 — Comparison of Normal Acceleration Equivalent Responses
F-14 Airplane — .5M/15,000 ft-Cockpit Control Position Input

TABLE V
F-14 AIRPLANE NORMAL ACCELERATION EQUIVALENT TRANSFER FUNCTIONS
Force Command Inputs

| Configuration | Airspeed (M/KEAS) | Match Variables | | | Equivalent System Parameters | | | | | | |
|----------------------|----------------------|---|-------|------------|---------------------------------|-----------------|------------|--------------|---------------|----------------|----------------|
| | | Response Parameter | Order | L_α | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch |
| CR ⁽¹⁾ | 0.5/248 | High Order System n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.3–10 ↓ | — | 0.773 | 0.61 | 2.78 | — | — |
| | | | 1/2 | Free | | 0.356 | — | 0.63 | 1.75 | 0.151 | 68.1 |
| | | | 0/2 | — | | 0.0264 0.364 | 0.773 | 0.60 0.60 | 1.81 1.81 | 0.165 0.153 | 60.4 71.3 |
| CR ⁽¹⁾ | 0.7/348 | High Order System n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.3–10 ↓ | — | 0.962 | 0.58 | 4.27 | — | — |
| | | | 1/2 | Free | | 1.219 | — | 0.74 | 2.75 | 0.14 | 36.1 |
| | | | 0/2 | — | | 0.0518 1.232 | 1.155 | 0.70 0.70 | 2.83 2.83 | 0.147 0.141 | 31.4 40.0 |
| CR ⁽¹⁾ | 0.83/412 | High Order System n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.3–10 ↓ | — | 0.965 | 0.62 | 5.01 | — | — |
| | | | 1/2 | Free | | 1.772 | — | 0.82 | 3.2 | 0.131 | 22.1 |
| | | | 0/2 | — | | 0.0639 1.779 | 1.116 | 0.79 0.79 | 3.26 3.26 | 0.137 0.132 | 19. 24.6 |
| CR ⁽¹⁾ | 1.2/596 | High Order System n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.3–10 ↓ | — | 1.143 | 0.71 | 6.88 | — | — |
| | | | 1/2 | Free | | 4.88 | — | 1.07 | 4.57 | 0.106 | 7.7 |
| | | | 0/2 | — | | 0.104 4.85 | 1.246 | 1.03 1.03 | 4.6 4.6 | 0.110 0.106 | 6.6 8.6 |
| PA ⁽¹⁾⁽²⁾ | 0.19/126 | High Order System ⁽³⁾ n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.1–10 ↓ | — | 0.473 | 0.52 | 1.48 | — | — |
| | | | 1/2 | Free | | 0.302 | — | 0.41 | 1.14 | 0.185 | 102.6 |
| | | | 0/2 | — | | 0.0068 0.310 | 0.612 | 0.38 0.38 | 1.19 1.19 | 0.182 0.186 | 91.3 112.2 |
| PA ⁽²⁾⁽²⁾ | 0.18/121 | High Order System ⁽³⁾ n_{zcr} $\dot{\theta}$ & n_{zcr} | 0/2 | — | 0.1–10 ↓ | — | 0.444 | 0.7 | 1.06 | — | — |
| | | | 1/2 | Free | | 0.241 | — | 0.49 | 0.81 | 0.183 | 108.2 |
| | | | 0/2 | — | | 0.0062 0.246 | 0.545 | 0.47 0.47 | 0.84 0.84 | 0.186 0.184 | 103.9 116.7 |

Notes: (1) 15,000 ft altitude

(2) Sea Level

(3) Phugoid component removed from high order system prior to performing match

A-7 Airplane — The A-7 airplane contains both a stability augmentation system which utilizes pitch rate and normal acceleration feedbacks, and a control augmentation system (CAS) which electrically feeds force commands forward to be summed with the mechanical command inputs. Both the normal acceleration and CAS signals are passed through a prefilter to eliminate high frequency inputs as outlined in appendix A. The high order pitch rate to pilot force command transfer function can be represented by a fourth order numerator over sixth order denominator. The A-7 equivalent system results are summarized in table VI.

Excellent equivalent system matches are obtained for the 0.6 and 0.9M cases as evidenced by the very low mismatch parameter. The 0.3M case on the other hand exhibits a large mismatch parameter attributable to the large separation in closed loop numerator and denominator roots introduced by the control system. At the higher speeds, these roots have migrated toward one another and have little influence on the resulting frequency response.

The response at 0.3 Mach is characterized by relatively high mismatch and galloping L_{α} ; similar to the F-14 control force command responses. Therefore, the normal acceleration response at the center of rotation, was included in the matching process, the results of which are included in table VI. As with the F-14 cases, the normal acceleration match parameters are consistent with those of the L_{α} -FIXED pitch rate analysis and simultaneous pitch rate and normal acceleration matching restrict the variation in L_{α} .

F-18 Airplane — The F-18 airplane possesses a highly complex digital flight control system. It incorporates numerous compensated feedbacks, stick shaping, lead-lag filters, etc., and has separate control law configurations for both cruise and power approach flight conditions as outlined in appendix A. The pitch rate response to control force inputs is described by a 14th order transfer function in cruise configuration and by an 11th order transfer function in power approach. Although the transfer functions are of relatively high order, the individual numerator and denominator roots in the short period frequency range are of similar magnitude and have only local influences on the total frequency response. As a result, very good match statistics are obtained for the low order systems as presented in table VII. There are, however, two highly damped oscillatory denominator root pairs in the cruise configuration — one arising from the aircraft's short period and one from the feedback network. It is difficult to determine, from the high order representation, how the combination of these roots will affect the aircraft's response and the pilot's opinion of it.

Matching the F-18's cruise configuration high order pitch rate response with the classical short period approximate system results in excellent agreement between the two systems. The resulting equivalent frequency ($\omega_{sp_e} = 3.05$ rad/sec) lies, on a Bode plot, midway between the two oscillatory pairs evident in the high order system.

The power approach configuration results are similar to those obtained for the other aircraft. Freeing L_{α} to improve the match statistics results in a good time history comparison but with a large value for L_{α} . The variation in L_{α} may be restricted by simultaneously matching pitch rate and normal acceleration.

TABLE VI
A-7 AIRPLANE EQUIVALENT TRANSFER FUNCTIONS⁽¹⁾

| Airspeed (M/KEAS) | Mixed Variables | | | Equivalent System Parameters | | | | | | |
|----------------------|---|-------------------|------------|---------------------------------|--------|------------|--------------|---------------|--------|----------|
| | Response Parameter | Order | L_α | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch |
| 0.3/149 | $\dot{\theta}$ $\ddot{\theta}$ n_{zcr} $\ddot{\theta}$ & n_{zcr} | High Order System | | 0.3-10 ↓ | — | 0.506 | 0.52 | 1.83 | — | — |
| | | 1/2 Fixed | | | 0.0145 | 0.506 | 0.42 | 1.37 | 0.10 | 78.7 |
| | | 1/2 Free | | | 0.012 | 1.484 | 0.31 | 1.77 | 0.077 | 41.0 |
| | | 0/2 — | | | 2.16 | — | 0.41 | 1.39 | 0.094 | 83.0 |
| | | 1/2 Free | | | 0.0137 | 0.696 | 0.39 | 1.43 | 0.093 | 67.0 |
| | | 0/2 — | | | 2.198 | — | 0.39 | 1.43 | 0.096 | 90.3 |
| 0.6/298 | $\dot{\theta}$ $\ddot{\theta}$ | High Order System | | 0.1-10 ↓ | — | 1.09 | 0.76 | 3.23 | — | — |
| | | 1/2 Fixed | | | 0.0482 | 1.09 | 0.75 | 2.90 | 0.061 | 9.3 |
| | | 1/2 Free | | | 0.0441 | 1.484 | 0.62 | 3.18 | 0.054 | 6.3 |
| 0.9/447 | $\dot{\theta}$ $\ddot{\theta}$ | High Order System | | 0.1-10 ↓ | — | 1.97 | 0.78 | 4.01 | — | — |
| | | 1/2 Fixed | | | 0.0846 | 1.97 | 0.87 | 5.34 | 0.034 | 1.1 |
| | | 1/2 Free | | | 0.0936 | 1.659 | 0.98 | 5.20 | 0.04 | 0.5 |

Note: (1) Cruise Configuration — 15,000 ft Altitude

TABLE VII
F-18 AIRPLANE EQUIVALENT TRANSFER FUNCTIONS

| Configuration | Match Parameters | | Equivalent System Parameters | | | | | |
|-------------------|--------------------------------------|---------------------------|------------------------------|-------------------|------------------|-------------------|------------|----------|
| | Response Parameter | Frequency Range (rad/sec) | K | L_α | ζ_{sp} | ω_{sp} | τ | Mismatch |
| CR ⁽¹⁾ | HOS $\dot{\theta}$ | 0.1-10 | — 0.0759 | 1.247 1.247(3) | 0.81/.76 1.01 | 1.11/4.47 3.05 | — 0.068 | — 1.5 |
| PA ⁽²⁾ | HOS ⁽⁴⁾ $\dot{\theta}$ | 0.1-10 ↓ | — | 0.395 | 0.68 | 1.78 | — | — |
| | $\dot{\theta}$ | | 0.0584 | 0.395(3) | 0.99 | 1.70 | 0.059 | 18.0 |
| | n_{zcr} | | 0.0518 | 0.736 | 0.69 | 2.06 | 0.047 | .5 |
| | $\dot{\theta}$ & n_{zcr} | | 0.122 | — | 0.94 | 1.62 | 0.08 | 23.8 |
| | | | 0.0543 | 0.442 | 0.93 | 1.69 | 0.052 | 15.7 |
| | | | 0.128 | — | 0.93 | 1.69 | 0.084 | 27.5 |

Notes: (1) .5M/10,000 ft altitude

(2) 133 KEAS, Sea Level

(3) L_α FIXED

(4) Phugoid component removed from HOS prior to performing match

MODIFIED EQUIVALENT SYSTEM DEFINITION

The equivalent systems obtained for the F-14 airplane's response to force commands were characterized by high mismatch values. Referring to the Bode plots presented in appendix B, this mismatch is most evident in the frequency range from 1 to 5 rad/sec. The cause of this mismatch may be traced to the single feel system denominator root at 3.37 rad/sec. Since there is no numerator root which approximates this term, it affects a large portion of the frequency response curves. Attempting to match such a high order system with the classical short period approximation restricts the quality of the match. There are not enough degrees of freedom for the search routine to account for the significant breakpoints in the frequency range of interest. Increasing the order of the equivalent system to account for such breakpoints should conceivably improve the match statistics.

The effect of accounting for significant components in the high order transfer function was investigated by matching the F-14 force command configurations with an equivalent transfer function of the form:

$$\frac{\dot{\theta}(s)}{F(s)} = \frac{K_{\theta} (s + L_{\alpha}) e^{-\tau s}}{(s^2 + 2\zeta_{sp} \omega_{sp} s + \omega_{sp}^2) (s + P_e)} \quad (5)$$

where P_e is the equivalent denominator root added to account for the uncompensated pole evident in the high order transfer function. The results of this matching process are presented in table VIII. Inclusion of the additional root in the equivalent system denominator greatly improved the mismatch parameter of the low order system; to the point that it is not necessary to even consider freeing L_{α} . The short period frequency is now greater than that obtained from the high order system's dominant roots. The time delay has been reduced to a value similar to that obtained from the control position analysis indicating that a large portion of the time delay associated with the first over second order equivalent models may be attributable to this additional feel system root.

The first over second and first over third order equivalent system responses are compared to the high order system in figure 6. The additional denominator root improves the transient response by eliminating the large time delay thereby improving the initial pitch acceleration characteristics. However, the assumed low order model is now inconsistent with that from which the specification requirements were generated. It would therefore seem necessary to determine new specification requirements which would account for the additional denominator root. This condition would, however, not be in concert with the initial desire to utilize existing specification formats and requirements.

COMPARISON WITH SPECIFICATION REQUIREMENTS

The equivalent system parameters obtained for each of the aircraft were compared against the requirements of MIL-F-8785C as summarized in table IX.

The damping ratio and time delay resulting from the equivalent system response can be compared directly against the requirements of MIL-F-8785C.

TABLE VIII
MODIFIED EQUIVALENT SYSTEM MODEL TRANSFER FUNCTIONS

| Airplane | Configuration | Airspeed (M/KEAS) | Match Variables | | | Equivalent System Parameters | | | | | | |
|----------|----------------------|----------------------|--|-------|---------------------------------|------------------------------|--------------------|--------------|---------------|----------------|--------------|--------------|
| | | | Transfer Function | Order | Frequency Range (rad/sec) | K | $L_{\alpha}^{(1)}$ | ζ_{sp} | ω_{sp} | τ | P_e | Mismatch |
| F-14 | CR ⁽²⁾ | 0.5/248 | High Order System $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.3-10 | — | 0.773 | 0.61 | 2.78 | — | — | — |
| | | | | 0/3 | | 0.159 2.066 | 0.773 — | 0.62 0.62 | 3.25 3.22 | 0.036 0.017 | 1.41 1.47 | 2.5 2.1 |
| | | 0.7/348 | High Order System $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.3-10 | — | 0.962 | 0.58 | 4.27 | — | — | — |
| | | | | 0/3 | | 0.334 7.47 | 0.962 — | 0.54 0.54 | 4.81 4.80 | 0.032 0.019 | 1.81 1.84 | 3.8 3.5 |
| | | 0.83/412 | High Order System $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.3-10 | — | 0.965 | 0.62 | 5.01 | — | — | — |
| | | | | 0/3 | | 0.437 11.62 | 0.965 — | 0.56 0.56 | 5.61 5.61 | 0.030 0.020 | 1.97 1.98 | 2.9 2.7 |
| | PA ⁽¹⁾⁽³⁾ | 1.2/596 | High Order System $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.3-10 | — | 1.143 | 0.71 | 6.88 | — | — | — |
| | | | | 0/3 | | 0.768 34.9 | 1.143 — | 0.59 0.60 | 7.97 7.99 | 0.020 0.013 | 2.25 2.26 | 3.0 2.9 |
| | | 0.19/126 | High Order System ⁽⁴⁾ $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.1-10 | — | 0.473 | 0.52 | 1.48 | — | — | — |
| | | | | 0/3 | | 0.0381 1.574 | 0.473 — | 0.53 0.53 | 1.46 1.46 | 0.043 0.04 | 2.83 2.83 | 0.02 0.02 |
| | | 0.18/121 | High Order System ⁽⁴⁾ $\dot{\theta}/F$ $n_{z_{cr}}/F$ | 1/3 | 0.1-10 | — | 0.444 | 0.7 | 1.06 | — | — | — |
| | | | | 0/3 | | 0.0335 1.245 | 0.444 — | 0.68 0.68 | 1.03 1.03 | 0.043 0.036 | 2.86 2.87 | 0.02 0.02 |

Notes: (1) L_{α} FIXED

(2) 15,000 ft Altitude

(3) Sea Level

(4) Phugoid component removed from HOS prior to performing match

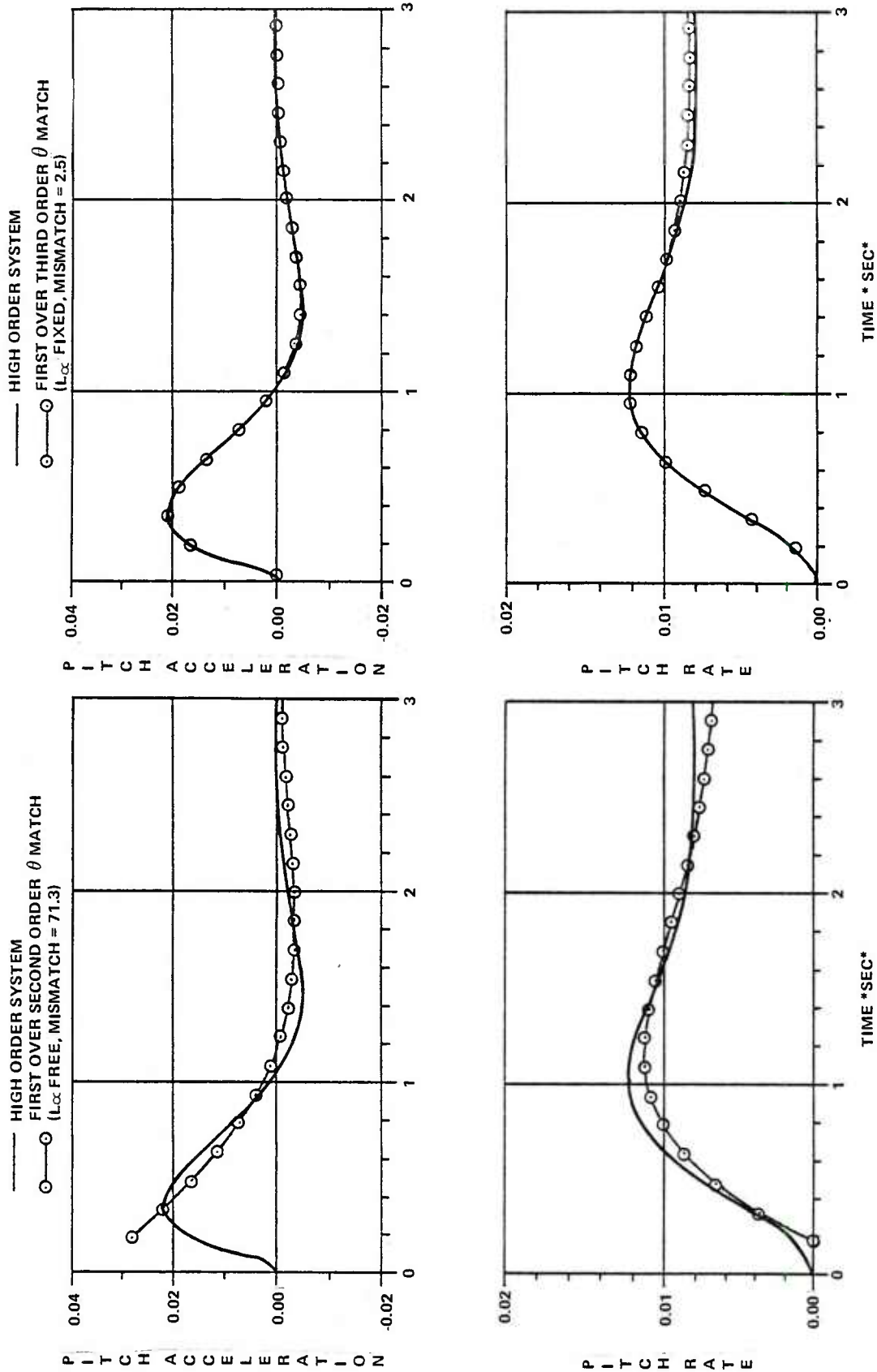


Figure 6 — Equivalent System Model Comparisons — F-14 Airplane — .5M/15,000 ft Altitude — Control Force Command Inputs

TABLE IX
MIL-F-8785C DYNAMIC LONGITUDINAL STABILITY REQUIREMENTS

| Level | Para. 3.2.2.1.1 $\omega_{sp}^2/n/\alpha$ | | | | Para. 3.2.2.1.2 ζ_{sp} | | Para. 3.5.3 τ | |
|--|---|------|----------------------|------|---------------------------------|-----|-----------------------|------|
| | Cat A | | Cat C | | Cat A & C | | All Categories | |
| | Min | Max | Min | Max | Min | Max | Min | Max |
| 1 | 0.28 ⁽¹⁾ | 3.6 | 0.16 ⁽³⁾ | 3.6 | 0.35 | 1.3 | 0 | 0.1 |
| 2 | 0.16 ⁽²⁾ | 10.0 | 0.096 ⁽⁴⁾ | 10.0 | 0.25 | 2.0 | 0.1 | 0.2 |
| 3 | 0.16 | — | 0.096 | — | 0.15 | — | 0.2 | 0.25 |
| Notes: (1) $\omega_{sp} > 1.0$ rad/sec for $n/\alpha < 3.5$ (2) $\omega_{sp} > 0.6$ rad/sec for $n/\alpha < 2.25$ (3) $\omega_{sp} > 0.85$ rad/sec for $2.5 < n/\alpha < 4.5$ (4) $\omega_{sp} > 0.6$ rad/sec for $1.6 < n/\alpha < 3.75$ | | | | | | | | |

In order to compare the short period frequency requirements against the specification, it is necessary to determine the equivalent acceleration sensitivity (n/α_e). It can be shown, under the assumption of constant speed equations, that for the steady state response to elevator inputs:

$$\frac{n}{\alpha} = \frac{\Delta n / \delta e}{\Delta \alpha / \delta e} = \frac{V}{g} \frac{1}{T_{\theta 2}} \approx \frac{V}{g} L_{\alpha} \quad (6)$$

Therefore, an equivalent n/α can be obtained by multiplying the equivalent L_{α} resulting from the matching process by V/g .

The parameters resulting from the equivalent system matches, along with the higher order system "dominant root" parameters, are compared against the specification requirements in figures 7 and 8 for configurations CR and PA, respectively. A number of observations can be made from reference to this data:

1. The frequency and damping ratio of the A-6 and S-3 "dominant" high order system roots are consistent with the equivalent system parameters. This may be attributed to the lack of control compensation roots in their high order systems.
2. The time delay parameter reflects level 1 flying qualities for all but the F-14 response to force command inputs in which level 2 time delays result.
3. Damping ratio is in the level 1 region for all conditions analyzed.
4. The A-6, S-3 and F-18 airplanes and the F-14's response to control position inputs all yield level 1 frequency characteristics. The frequency value obtained from the dominant root, L_{α} -fixed and L_{α} -free analyses may, however, be significantly different.
5. The A-7 and the F-14 responses to control force inputs not only result in different values of frequency for the differing analysis techniques, but the identified parameters tend to cross over flying qualities boundaries. For example, the dominant F-14 root at 0.5M/15,000 feet indicates level 1 frequency characteristics, the $\dot{\theta}$ - L_{α} fixed and simultaneous $\dot{\theta}$ and n_z equivalent systems indicate level 2-3 characteristics, and the $\dot{\theta}$ - L_{α} free case provides an indication that the handling qualities are, in this instance, worse than level 3.

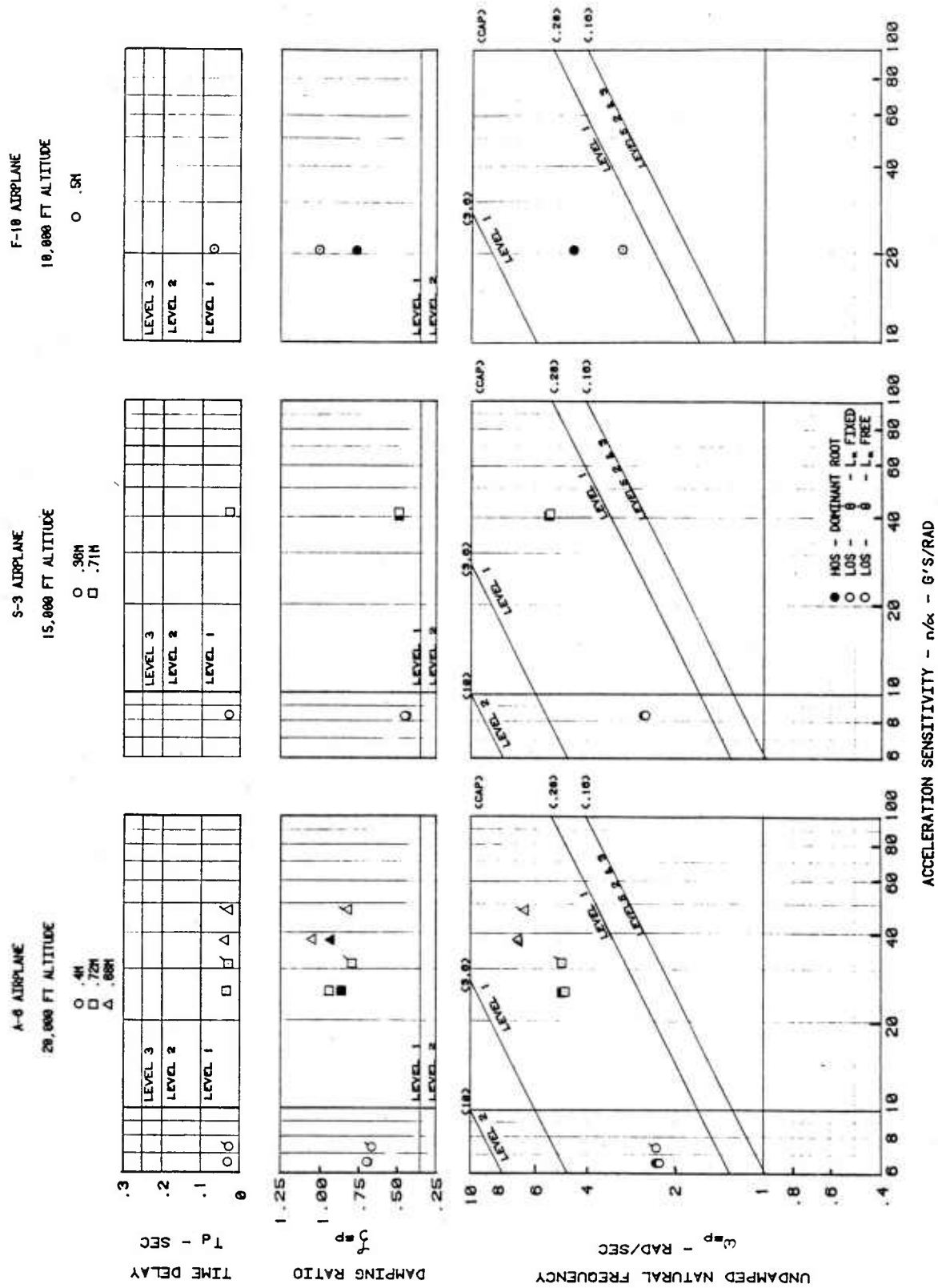
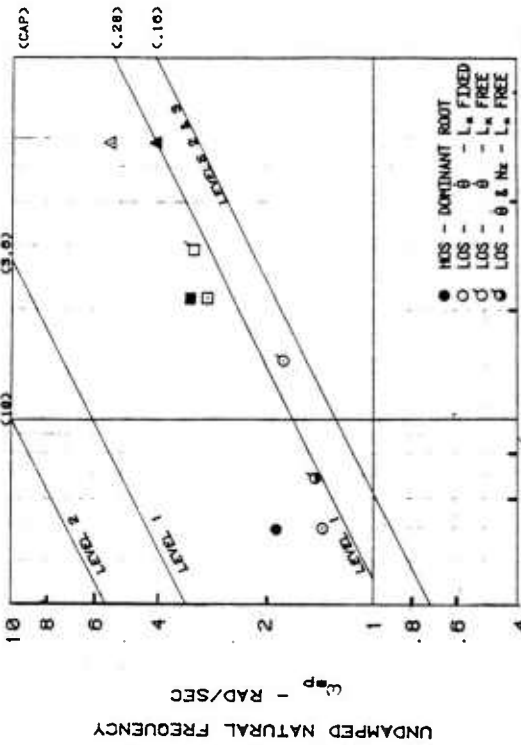
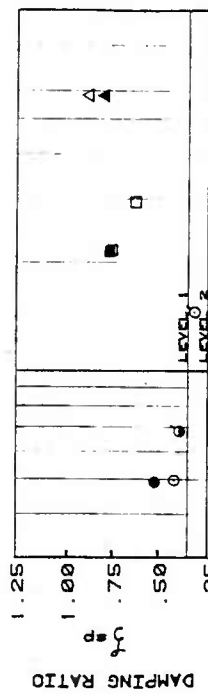
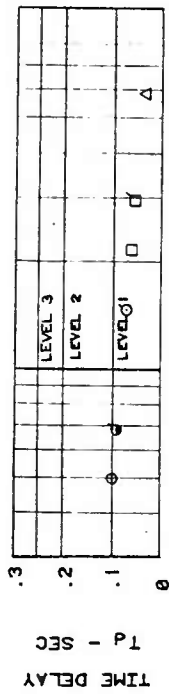


Figure 7 – Cruise Configuration Specification Compliance

A-7 AIRPLANE

15,000 FT ALTITUDE

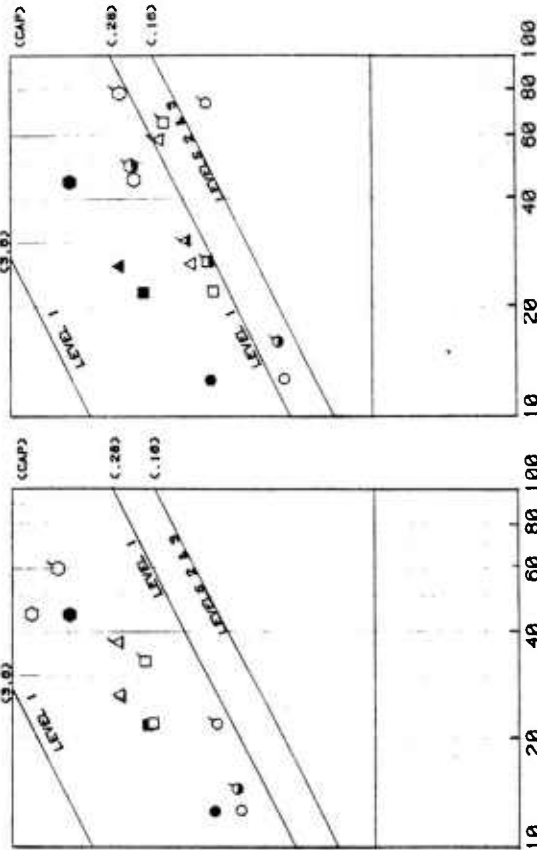
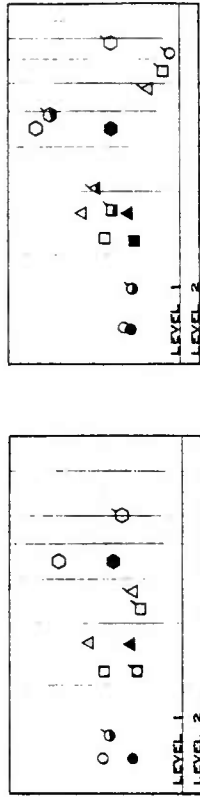
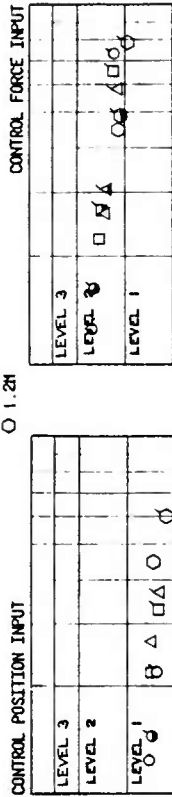
○ .3M
□ .6M
△ .9M



F-14 AIRPLANE

15,000 FT ALTITUDE

○ .5M
□ .7M
△ .83M
○ 1.2M



ACCELERATION SENSITIVITY - α/α_0 - G'S/RAD

Figure 7. (continued) — Cruise Configuration Specification Compliance

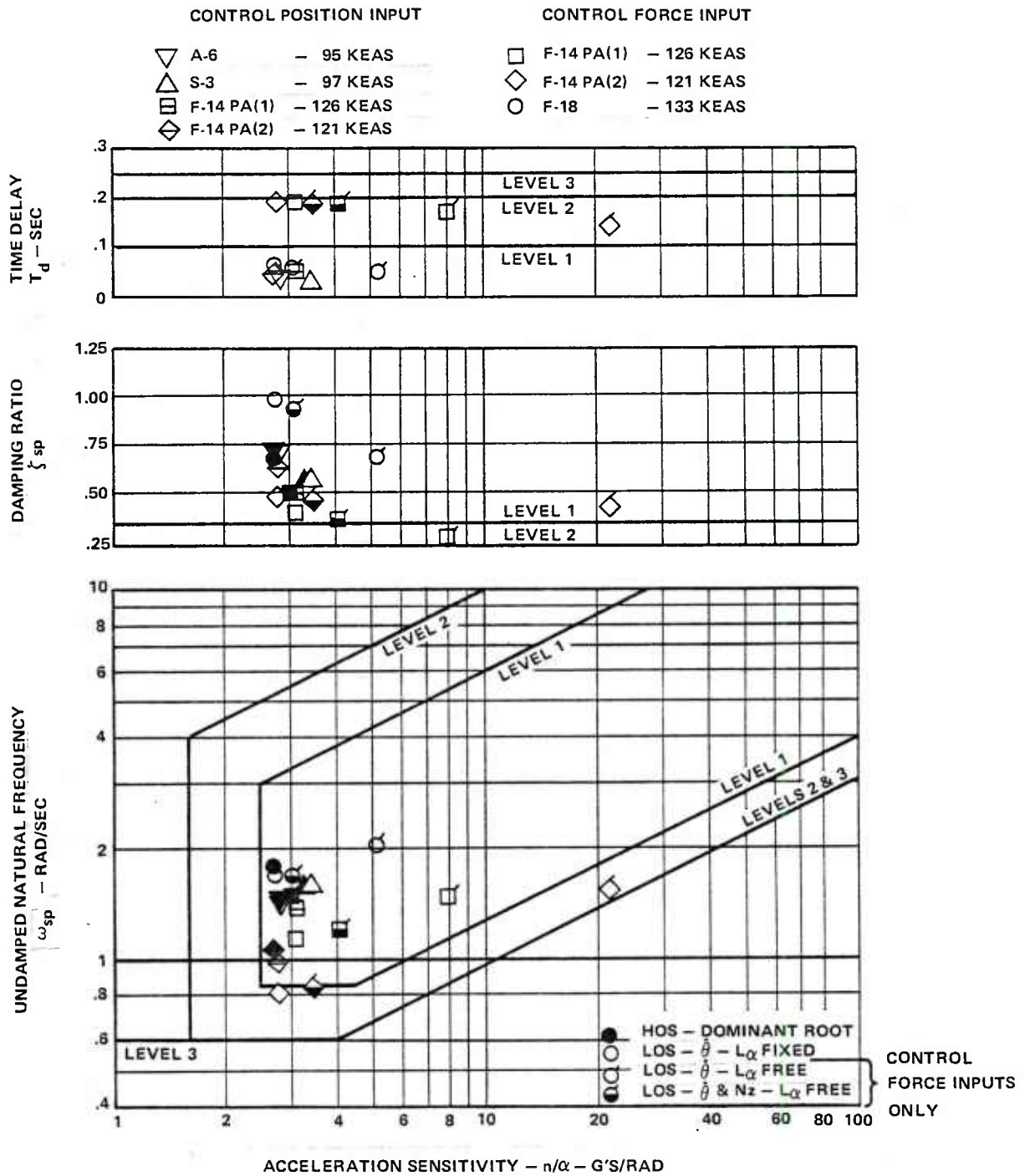


Figure 8 - Power Approach Specification Compliance

The basic premise under which equivalent systems have been developed hypothesizes that: an equivalent system which has similar dynamics (i.e. frequency and time history characteristics) as a higher order system will be similarly evaluated by the pilot during maneuvering tasks. However, the final observation made above indicates differences in flying qualities levels and therefore pilot acceptability for various low order descriptions of a particular high order model. The frequency and time history comparisons (Appendix B) and their respective mismatch parameters are within reasonable bounds as defined in previous investigations. (It was determined, from the flight tests of reference q that configurations with mismatch parameters as high as 200 were not noticeable to pilots when evaluating their equivalence.) Therefore, it becomes necessary to further investigate the short period frequency specification methodology of MIL-F-8785C.

CONTROL ANTICIPATION PARAMETER CORRELATION

The longitudinal short period frequency requirements of MIL-F-8785C are not only presented as a function of n/α but are also correlated with pilot opinion (i.e. flying qualities levels) by the control anticipation parameter (CAP). Bihle (reference r) defined CAP as relating the two responses of primary interest to the pilot during a pullup — the initial pitch acceleration and the steady state normal acceleration. He further showed that for the constant speed short period approximation:

$$CAP = \frac{\ddot{\theta}(t=0^+)}{n_{z_{ss}}} = \frac{\omega_{sp}^2}{n/\alpha} \quad (7)$$

This parameter provides boundaries on pilot opinion of acceptable short period frequency characteristics when presented in the format of MIL-F-8785C.

The equivalent system model is the same as the short period approximation. Equation (7) is, therefore, identically correct for the equivalent system model. It is possible to plot ω_{sp_e} vs n/α_e and correlate it with an equivalent control anticipation parameter, CAP_e . However, in the case of higher order systems, it has been shown by DiFranco, reference (s), that the initial pitch acceleration is zero and builds to a maximum at some time greater than $t = 0^+$. In this instance, the pitch acceleration of interest in defining the control anticipation parameter is the maximum pitch acceleration and not the initial pitch acceleration. The control anticipation parameter is then defined as:

$$CAP' = \frac{\ddot{\theta}_{max}}{n_{z_{ss}}} = \frac{\omega_{sp}^2}{n/\alpha} \ddot{\theta}_{nd} \quad (8)$$

where $\ddot{\theta}_{nd}$ is a non-dimensional pitch acceleration. It is the ratio of pitch acceleration, including the high order control system components, to the pitch acceleration at time $t = 0^+$ excluding these same control components, following a step control input.

Comparing equations (7) and (8) it can be seen that the equivalent system CAP will not, in general, be the same as the high order system CAP' . Although the equivalent system parameters can be correlated unto themselves, they will not correlate with the high order system parameters which they are being touted to represent. However, it was noted in reference (q) that the equivalent pitch acceleration has characteristics similar to Di Franco's nondimensionalizing pitch acceleration. Therefore, an equivalent system attenuation factor can be defined as the ratio of the maximum pitch acceleration of the high order system to the initial acceleration of the low order system, which will correlate the two representations. It was also shown in reference (s), using available inflight research results, that the CAP' boundaries of table X can be utilized to correlate high order responses with pilot opinion ratings.

TABLE X
ATTENUATED CONTROL ANTICIPATION PARAMETER BOUNDARIES – CATEGORY A

| Level | CAP' | |
|-------|------|-----|
| | Min | Max |
| 1 | 0.25 | 1.5 |
| 2 | 0.15 | — |

Two examples will serve to illustrate the differences in utilizing CAP' as opposed to CAP to correlate the equivalent short period responses. The parameters of interest for two selected cases are summarized in table XI.

TABLE XI
CAP AND CAP' COMPARISONS

| Parameter | A-6 Airplane – $\dot{\theta}/\delta_{ep}$ 0.72M/20,000 ft altitude | | | F-14 Airplane – $\dot{\theta}/F$ 0.5M/15,000 ft altitude | | |
|------------------------------------|---|--------------------------|-------------------------|---|--------------------------|-------------------------|
| | HOS | LOS | | HOS | LOS | |
| | | $L_{\alpha\text{FIXED}}$ | $L_{\alpha\text{FREE}}$ | | $L_{\alpha\text{FIXED}}$ | $L_{\alpha\text{FREE}}$ |
| ω_{sp} | 4.86 | 4.75 | 4.84 | 2.78 | 1.74 | 2.88 |
| n/α | 24.8 | 25.0 | 31.2 | 12.6 | 12.7 | 73.6 |
| $\ddot{\theta}(0^+)$ | 0 | 0.507 | 0.444 | 0 | 0.0278 | 0.0172 |
| $\ddot{\theta}_{\max}$ | 0.305 | 0.507 | 0.444 | 0.021 | 0.0278 | 0.0172 |
| n_{zss} | 0.612 | 0.561 | 0.590 | 0.141 | 0.117 | 0.153 |
| $\omega_{sp}^2/n/\alpha$ | 0.952 | 0.903 | 0.751 | 0.613 | 0.238 | 0.113 |
| $CAP = \ddot{\theta}(0^+)/n_{zss}$ | 0 | 0.904 | 0.753 | 0 | 0.238 | 0.113 |
| CAP' | 0.498 | 0.544 | 0.517 | 0.150 | 0.182 | 0.139 |

From the high order system data, it can be seen that the classical definition of CAP provides no information about the resulting response since the initial acceleration is zero. If the aircraft short period root is utilized to calculate CAP, false indications again result since the attenuation effects of the control system are not accounted for. Additionally, the equivalent system models yield differing CAP values for the L_{α} -fixed and free cases, neither of which are equal to $\omega_{sp}^2/n/\alpha$ of the high order system.

The premise under which the short period frequency requirements are established indicate that as acceleration sensitivity is increased, short period frequency should also be increased to maintain constant pilot rating (i.e. constant control anticipation parameter). It could therefore be assumed that for the low order system to be rated by a pilot as being equivalent to the high order system, this same relationship should hold. However, as L_{α} is freed in the search routine, the CAP value is reduced. Such observations would tend to discount the equivalent system procedures. However, if the differences in the pitch acceleration characteristics are accounted for via an attenuation factor, very good agreement is found among the example cases.

Referring back to figures 7 and 8, there is no constant relationship evident between each of the system models and CAP. The data necessary to define CAP and CAP' for each of the conditions analyzed in this report are presented in tables XII and XIII. Very good agreement between CAP' and CAP'_e is seen to exist for all cases with the largest variation in the two parameters arising from the configurations with the highest mismatch.

The data may be viewed in the format of MIL-F-8785C by determining an attenuated or effective frequency for which CAP' is constant. From equation (8) this effective frequency can be defined as:

$$\omega_{\text{EFF}} = \omega_{\text{sp}} \sqrt{\ddot{\theta}_{\text{nd}}} = \sqrt{(\text{CAP}') (n/\alpha)} \quad (9)$$

This parameter is most easily obtained for the high order system from the relationship:

$$\omega_{\text{EFF}} = \sqrt{\left(\frac{\ddot{\theta}_{\text{max}}}{n z_{\text{ss}}} \right)} n/\alpha \quad (10)$$

while for the equivalent systems

$$\omega_{\text{EFF}} = \omega_{\text{sp}_e} \sqrt{\frac{\ddot{\theta}_{\text{max HOS}}}{\ddot{\theta}_{\text{LOS}} (t = \tau)}} \quad (11)$$

This effective or attenuated short period frequency is plotted as a function of n/α in figures 9 and 10. The level 1 and 2 boundaries for configuration CR are those presented in table X. CAP' boundaries have not been determined for approach configurations. Reference (d) indicates that Category A (maneuvering) boundaries are applicable to the touchdown portion of a landing, and Category C (approach) boundaries are applicable to the approach phase. Therefore, as a point of reference in defining constant CAP', the Category A boundaries of table X were included in figure 10.

The data presented in figures 9 and 10 not only indicate that CAP' provides an additional measure of consistency of equivalent systems, but that CAP' provides a more accurate representation of frequency characteristics for advanced aircraft than does CAP. The effective frequency data for each flight condition analyzed lies along a line of nearly constant CAP' for all of the various search methods (including the dominant root analysis). As a result, a particular level of flying qualities can now be identified for each flight condition. (Unfortunately, no pilot opinion data obtained during maneuvering tasks is available to verify the identified levels).

Comparing figures 7 and 9 and figures 8 and 10, interesting trends can be noted in the identified level of flying qualities. Those aircraft for which the control system does not significantly affect the frequency characteristics (A-6 and S-3) indicate solid level 1 flying qualities whether compared against CAP or CAP'. However, the other aircraft, in which the control system attenuates the response, are nearer the lower frequency boundary when correlated with CAP' rather than CAP. Since CAP' is calculated from the aircraft responses (pitch and normal acceleration), it is interpreted as more accurately representing the aircraft's characteristics.

The utilization of CAP' also provides the opportunity for analyzing the results of other than first over second order equivalent systems with regard to specification requirements. It was shown in reference (s) that for systems with higher order poles and zeros, CAP' can be defined as:

TABLE XII
CONTROL ANTICIPATION PARAMETER DATA — CRUISE CONFIGURATION

| Aircraft | Altitude | Mach | High Order System | | | System Match | | Low Order System | | | | | |
|----------|----------|------|-------------------|-----------------------|-----------|--------------|------------------------------|------------------|------------|--------------|------------------|-------------------|----------|
| | | | n/α | $\dot{\theta}_{\max}$ | n_{zss} | CAP' | Parameter | Order | L_α | n/α_e | CAP _e | CAP' _e | Mismatch |
| A-6 | 20,000 | 0.4 | 6.52 | 0.108 | 0.163 | 0.663 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 6.51 | 0.793 | 0.640 | 1.8 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 7.27 | 0.734 | 0.602 | 1.2 |
| | | | 24.8 | 0.305 | 0.612 | 0.499 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 25.0 | 0.903 | 0.543 | 2.8 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 31.2 | 0.751 | 0.516 | 0.5 |
| S-3 | 15,000 | 0.88 | 37.3 | 0.41 | 0.794 | 0.516 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 37.7 | 1.241 | 0.588 | 3.6 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 47.7 | 0.894 | 0.56 | 1.2 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 8.37 | 0.777 | 0.659 | 1.8 |
| | | | 8.35 | 6.244 | 9.23 | 0.677 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 41.1 | 0.707 | 0.540 | 0.25 |
| F-14 | 15,000 | 0.71 | 40.4 | 17.33 | 32.10 | 0.539 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 12.7 | 0.439 | 0.317 | 10.9 |
| | | | 12.6 | 0.200 | 0.701 | 0.285 | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 21.9 | 0.343 | 0.28 | 0.6 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 22.1 | 0.753 | 0.469 | 11.4 |
| | | | 21.9 | 0.388 | 0.959 | 0.405 | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 33.2 | 0.560 | 0.421 | 0.8 |
| F-14 | 15,000 | 0.83 | 26.0 | 0.478 | 1.002 | 0.477 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 26.3 | 0.954 | 0.546 | 8.7 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 36.7 | 0.700 | 0.496 | 0.4 |
| | | | 44.5 | 0.797 | 1.313 | 0.607 | $\dot{\theta}/\delta ep$ | 1/2 | FIX | 45.1 | 1.655 | 0.690 | 6.8 |
| | | | | | | | $\dot{\theta}/\delta ep$ | 1/2 | FREE | 58.5 | 0.961 | 0.639 | 0.9 |
| F-14 | 15,000 | 0.5 | 12.6 | 0.021 | 0.141 | 0.150 | $\dot{\theta}/Fx$ | 1/2 | FIX | 12.7 | 0.238 | 0.182 | 71.3 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 73.6 | 0.112 | 0.139 | 11.4 |
| | | | | | | | $\dot{\theta} n_{z_{cr}}/Fx$ | 1/2,0/2 | FREE | 16.1 | 0.204 | 0.162 | 60.4 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/3 | FIX | 12.7 | 0.832 | 0.155 | 2.5 |
| F-14 | 15,000 | 0.7 | 21.9 | 0.033 | 0.192 | 0.172 | $\dot{\theta}/Fx$ | 1/2 | FIX | 22.1 | 0.342 | 0.207 | 37.3 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 64.6 | 0.206 | 0.173 | 6.6 |
| | | | | | | | $\dot{\theta} n_{z_{cr}}/Fx$ | 1/2,0/2 | FREE | 26.6 | 0.301 | 0.192 | 31.4 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/3 | FIX | 22.1 | 1.047 | 0.187 | 3.8 |
| F-14 | 15,000 | 0.83 | 26.0 | 0.037 | 0.201 | 0.184 | $\dot{\theta}/Fx$ | 1/2 | FIX | 26.3 | 0.387 | 0.214 | 22.7 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 58.3 | 0.258 | 0.185 | 4.7 |
| | | | | | | | $\dot{\theta} n_{z_{cr}}/Fx$ | 1/2,0/2 | FREE | 30.4 | 0.35 | 0.202 | 19.0 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/3 | FIX | 26.3 | 1.197 | 0.199 | 2.9 |
| F-14 | 15,000 | 1.2 | 44.5 | 0.051 | 0.263 | 0.194 | $\dot{\theta}/Fx$ | 1/2 | FIX | 45.1 | 0.461 | 0.218 | 7.9 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 75.7 | 0.322 | 0.200 | 0.9 |
| | | | | | | | $\dot{\theta} n_{z_{cr}}/Fx$ | 1/2,0/2 | FREE | 49.1 | 0.431 | 0.211 | 6.6 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FIX | 45.1 | 1.408 | 0.210 | 3.0 |
| F-14 | 15,000 | 0.3 | 4.93 | 0.012 | 0.044 | 0.273 | $\dot{\theta}/Fx$ | 1/2 | FIX | 4.98 | 0.377 | 0.312 | 78.7 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 14.6 | 0.215 | 0.215 | 41.0 |
| | | | | | | | $\dot{\theta} n_{z_{cr}}/Fx$ | 1/2,0/2 | FREE | 6.85 | 0.299 | 0.261 | 67.0 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/3 | FIX | 4.98 | 0.567 | 0.266 | 5.6 |
| F-18 | 10,000 | 0.6 | 21.3 | 0.031 | 0.118 | 0.263 | $\dot{\theta}/Fx$ | 1/2 | FIX | 21.5 | 0.391 | 0.252 | 9.3 |
| | | | | | | | $\dot{\theta}/Fx$ | 1/2 | FREE | 29.3 | 0.345 | 0.243 | 6.3 |
| | | | 58.0 | 0.053 | 0.172 | 0.308 | $\dot{\theta}/Fx$ | 1/2 | FIX | 58.2 | 0.490 | 0.307 | 1.1 |
| | | | 20.7 | 0.058 | 0.175 | 0.330 | $\dot{\theta}/Fx$ | 1/2 | FIX | 20.8 | 0.447 | 0.342 | 1.5 |

TABLE XIII
CONTROL ANTICIPATION PARAMETER DATA — Power Approach Configuration

| Aircraft | Airspeed (KEAS) | High Order System | | | System Match | | | Low Order System | | | | |
|---------------------|--------------------|-------------------|------------------------|-----------|--------------|------------------------------|---------|---------------------|--------------|------------------|-------------------|----------|
| | | n/α | $\ddot{\theta}_{\max}$ | n_{zss} | CAP' | Parameter | Order | L_{α} | n/α_e | CAP _e | CAP' _e | Mismatch |
| A-6 | 95 | 2.82 | 0.112 | 0.181 | 0.621 | $\dot{\theta}/\delta_{ep}$ | 1/2 | FIX ⁽¹⁾ | 2.30 | 0.701 | 0.581 | 5.5 |
| | | | | | | $\ddot{\theta}/\delta_{ep}$ | 1/2 | FREE ⁽¹⁾ | 4.26 | 0.557 | 0.479 | 2.4 |
| | | | | | | $\ddot{\theta}/\delta_{ep}$ | 1/2 | FIX | 2.88 | 0.722 | 0.63 | 0.2 |
| S-3 | 97 | 3.33 | 1.921 | 3.214 | 0.6 | $\dot{\theta}/\delta_{ep}$ | 1/2 | FIX ⁽¹⁾ | 3.03 | 0.675 | 0.592 | 3.1 |
| | | | | | | $\ddot{\theta}/\delta_{ep}$ | 1/2 | FIX | 3.43 | 0.664 | 0.601 | 0.1 |
| F-14 ⁽²⁾ | 126 | 3.05 | 0.051 | 0.098 | 0.519 | $\dot{\theta}/\delta_{ep}$ | 1/2 | FIX | 3.13 | 0.609 | 0.529 | 2.1 |
| F-14 ⁽³⁾ | 121 | 2.74 | 0.046 | 0.156 | 0.292 | $\dot{\theta}/\delta_{ep}$ | 1/2 | FIX | 2.82 | 0.334 | 0.296 | 2.0 |
| F-14 ⁽²⁾ | 126 | 3.05 | 0.0068 | 0.020 | 0.345 | $\dot{\theta}/F$ | 1/2 | FIX | 3.13 | 0.416 | 0.387 | 102.7 |
| | | | | | | $\ddot{\theta}/F$ | 1/2 | FREE | 8.02 | 0.273 | 0.310 | 61.5 |
| | | | | | | $\dot{\theta}, n_{z_{cr}}/F$ | 1/2,0/2 | FREE | 4.05 | 0.350 | 0.350 | 91.3 |
| F-14 ⁽³⁾ | 121 | 2.74 | 0.0065 | 0.031 | 0.207 | $\ddot{\theta}/F$ | 1/3 | FIX | 3.13 | 0.681 | 0.344 | 0.02 |
| | | | | | | $\dot{\theta}/F$ | 1/2 | FIX | 2.82 | 0.233 | 0.233 | 108.5 |
| | | | | | | $\ddot{\theta}/F$ | 1/2 | FREE | 21.94 | 0.108 | 0.172 | 46.2 |
| F-18 | 133 | 2.72 | 0.047 | 0.064 | 0.735 | $\dot{\theta}, n_{z_{cr}}/F$ | 1/2,0/2 | FREE | 3.46 | 0.204 | 0.214 | 103.9 |
| | | | | | | $\ddot{\theta}/F$ | 1/3 | FIX | 2.82 | 0.376 | 0.209 | 0.02 |
| | | | | | | $\dot{\theta}/F$ | 1/2 | FIX | 2.76 | 1.047 | 0.839 | 18.0 |
| | | | | | | $\ddot{\theta}/F$ | 1/2 | FREE | 5.13 | 0.826 | 0.747 | 0.5 |
| | | | | | | $\dot{\theta}, n_{z_{cr}}/F$ | 1/2,0/2 | FREE | 3.08 | 0.926 | 0.798 | 15.7 |

Notes: (1) Phugoid contribution included in high order system

(2) Configuration PA(1)

(3) Configuration PA(2)

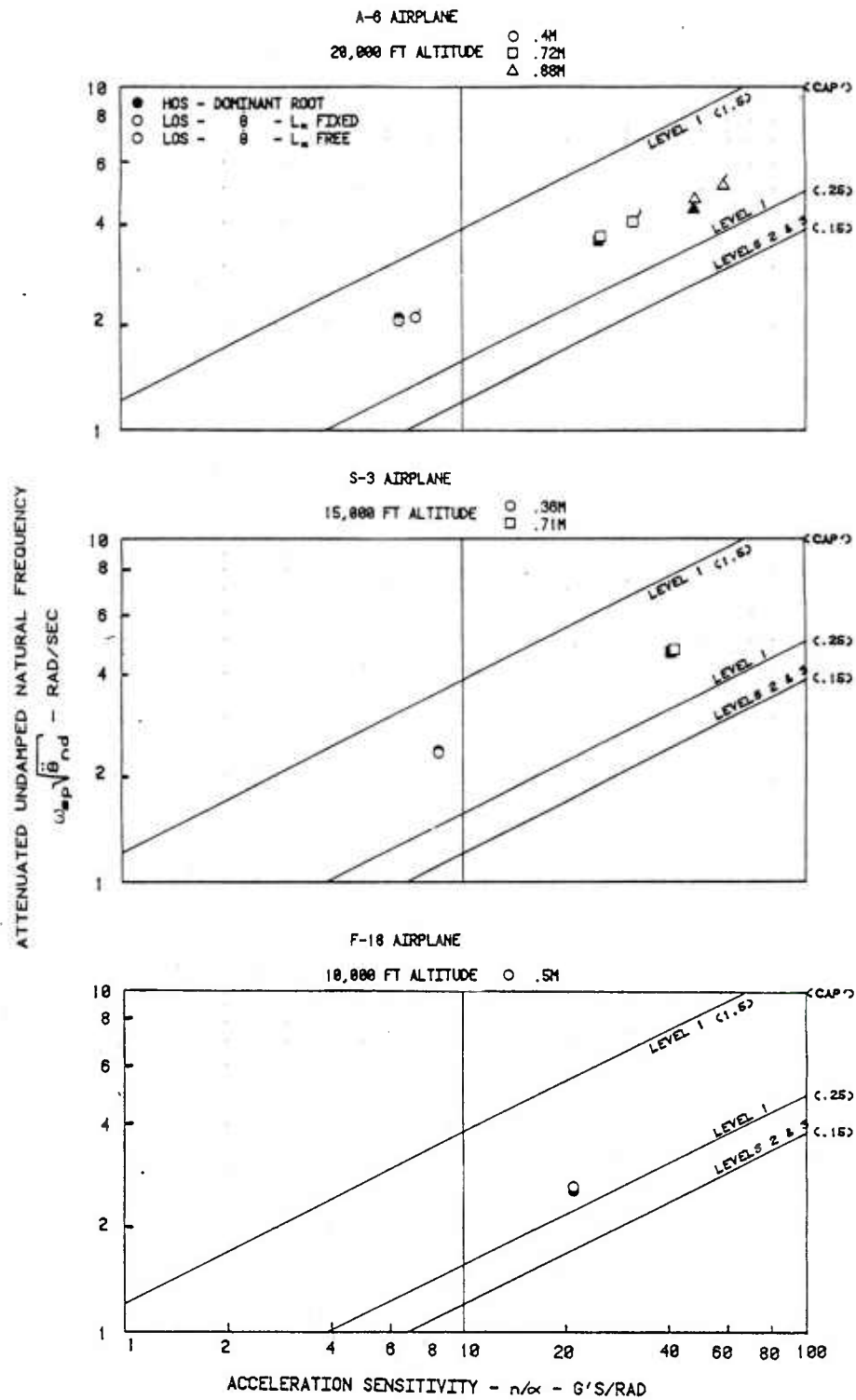


Figure 9 - CAP' Correlation of Cruise Configuration Short Period Frequency Characteristics

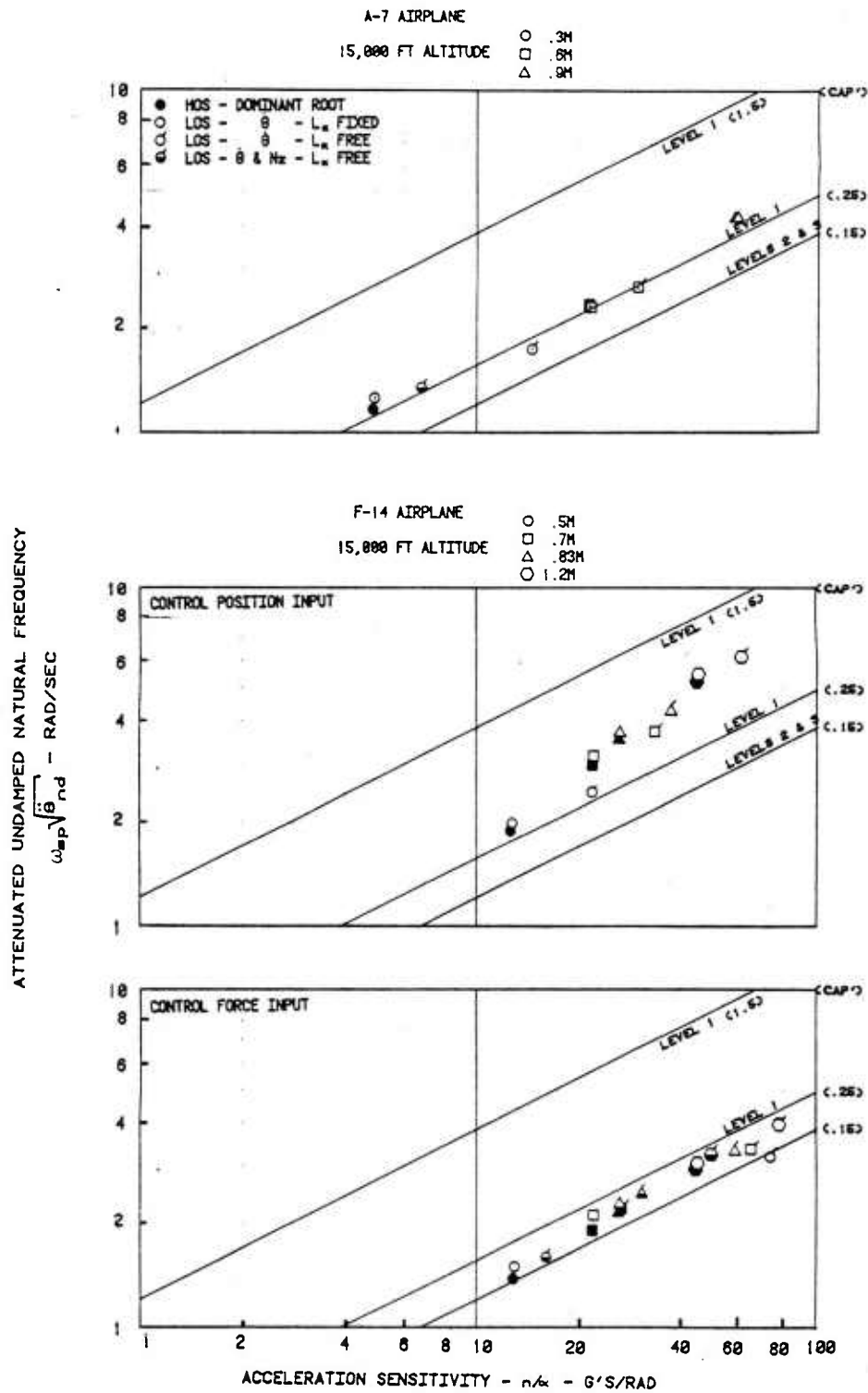


Figure 9 (continued) — CAP' Correlation of Cruise Configuration Short Period Frequency

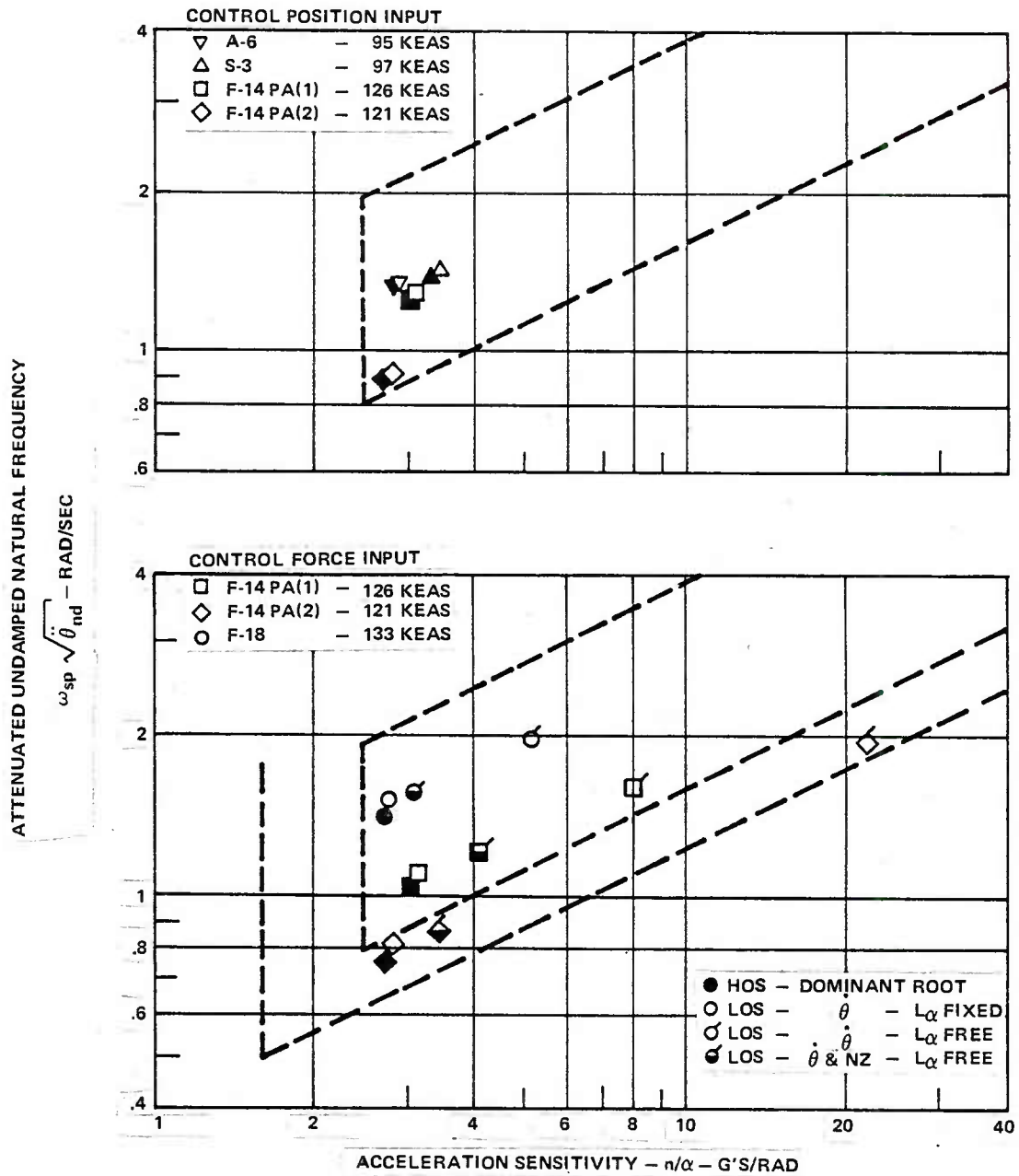


Figure 10 - CAP' Correlation of Power Approach Configuration Short Period Frequency Characteristics

$$CAP'_e = \frac{\omega_{sp}^2}{n/\alpha} \frac{\ddot{\theta}_{\max} \text{HOS}}{\ddot{\theta}(t=\tau)_{\text{LOS}}} \cdot \frac{\pi P_e}{\pi z_e}, \quad (12)$$

where P_e and z_e represent the poles and zeros, respectively, necessary to improve the response match in the frequency range of interest. Equation (12) was utilized to compute CAP'_e for the F-14 and A-7 cases in which first over third order equivalent systems were determined. The results for each of these cases are included in tables XII and XIII. CAP'_e for each of the first over third order matches showed better correlation with the high order system CAP' than did the first over second order equivalent systems. This improvement in CAP'_e can also be correlated directly with the mismatch parameter — as mismatch is improved so is the high order vs low order CAP' . The attenuated control anticipation parameter can, then, be used as a method of correlating increased order equivalent systems with specification frequency requirements.

CONCLUSIONS

Longitudinal short period equivalent models have been determined which represent the high order dynamics of five tactical Navy aircraft. The frequency response matching method utilized was straightforward and easy to implement. The methods applied in determining the best low order system matches can be divided into three categories based on their frequency content: low, medium, and high.

- Low frequency components were essentially ignored in the matching process. Where the identified equivalent short period roots were closely coupled to the ignored low frequency roots (power approach configurations) it was necessary to completely remove their contribution from the high order representation in order to obtain the best modal parameter correlation.
- High frequency components were adequately modelled by a time delay parameter.
- Mid frequency components, in the pilot's frequency range of interest, compound the matching process. When numerator and denominator roots of approximately equal magnitude are introduced in the high order system, the matching process easily identifies a set of equivalent system roots. When separation occurs between the numerator and denominator roots, or an unequal number of roots are included, the frequency matching technique identifies an apparent numerator root for the classical first over second order short period approximation, which may be significantly different from the L_α of the airplane. This condition may be alleviated by either 1) simultaneously matching pitch rate and normal acceleration and accepting the (relatively) large mismatch or 2) introducing additional roots into the equivalent system model to account for the dominating high order system components.

Correlation of the equivalent system modal parameters with MIL-SPEC requirements was straightforward for both short period damping ratio and time delay. Correlation of short period frequency characteristics for those aircraft with significant control compensation, however, showed variation in identified flying qualities levels. The level of flying qualities for the high order system was different from the equivalent solution with L_α -fixed and from the L_α -free solution when analyzed via the traditional control anticipation parameter. These results led to the definition of a control anticipation parameter which allows the correlation of both high order and equivalent low order systems to pilot opinion ratings and hence to specification boundaries. This attenuated control anticipation parameter not only correlates the classical short period equivalent system model with the physical characteristics of the high order system, but provides the opportunity for comparing alternate low order equivalent system forms with flying qualities requirements.

The A-6, A-7, S-3, and F-18 aircraft responses all resulted in level 1 flying qualities for each of the parameters analyzed, at all flight conditions. The A-7 frequency response was, however, only marginally level 1, with a number of data points lying directly on the boundary. The F-14 airplane exhibited level 1 performance as indicated by the response to control position commands. However, in response to force inputs, the F-14 airplane would be predicted to be a level 2 airplane both in cruise and non-DLC power approach conditions. This characteristic is evidenced by the high time delay obtained from the first over second order equivalent analysis and from the effective frequency characteristics as correlated with CAP'. This apparent degradation in the F-14's handling qualities can be traced to the feel system implementation.

RECOMMENDATIONS

Frequency response matching techniques provide an effective means of obtaining low order equivalent models of high order longitudinal aircraft responses. When using the classical short period approximation as the equivalent system model, pitch rate and normal acceleration should be matched simultaneously, constraining the characteristic denominator roots to be equal, to reduce the variation in L_α — the numerator root.

The ratio of the maximum pitch acceleration (rather than the initial short period component) to the steady state normal acceleration (CAP') experienced in a pullup should be utilized as the correlating parameter for MIL-SPEC frequency characteristics. Additional work should be performed to verify the Category A CAP' boundaries utilized in this report and to establish similar Category C boundaries.

APPENDIX A
AIRCRAFT AND CONTROL SYSTEM
DESCRIPTIONS

Data describing each of the airplanes and their respective control systems were obtained from available aerodynamic stability and control reports, references (h) through (m). This appendix briefly describes the subject airplanes and presents a block diagram of their respective longitudinal control system as modelled in this analysis.

A-6 — The A-6 airplane is a twin turbojet, land and carrier based, subsonic, all-weather attack aircraft. Longitudinal control is transmitted from the pilot's control stick, via bellcranks and pushrods, to an all-moving horizontal stabilizer. The control stick feel system and bobweight arrangement was not included in the present model. The basic aircraft stability is augmented by the feedback of washed out pitch rate to the horizontal stabilizer. A simplified block diagram of the A-6 airplane's longitudinal control system, as modelled in this analysis is presented in figure A-1.

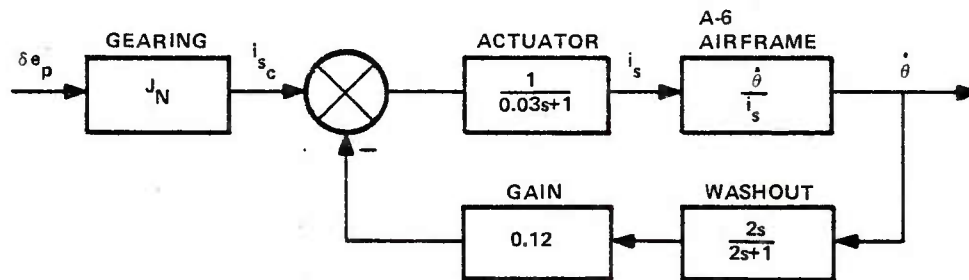


Figure A-1 — A-6 Airplane Longitudinal Control System Block Diagram

The A-6 airplane's pitch rate response to pilot longitudinal control inputs can be represented by a fourth order numerator over sixth order denominator transfer function of the form:

$$\frac{\dot{\theta}(s)}{\delta e_p(s)} = \frac{(J_N) N_{\dot{\theta}} (2s + 1)}{\Delta (2s + 1) (.03s + 1) + N_{\dot{\theta}} (.24s)} \quad (a-1)$$

where $J_N = 0.0122$ rad/in in cruise configuration and 0.0232 rad/in in power approach configuration. The transfer functions representing the A-6 airplane, as analyzed in this report, are presented in table A-1.

S-3 — The S-3A airplane is a twin turbofan powered, land and carrier based, subsonic, anti-submarine warfare aircraft. Longitudinal control is accomplished via a mechanical control system which operates the elevator. Control stick dynamics were not included in the model of the S-3 aircraft. There is no longitudinal stability augmentation system included in the aircraft. A block diagram of the S-3's control system, as modelled in this analysis, is presented in figure A-2.

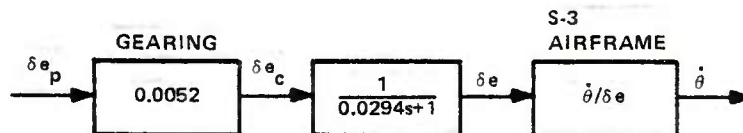


Figure A-2 — S-3 Airplane Longitudinal Control System Block Diagram

TABLE A-I. A-6 AIRPLANE TRANSFER FUNCTIONS

| Config- uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|--------------------|------------------|----------------------|--|
| CR | 20,000 | 0.4/179 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{4.31 (0) (.0147) (.506) (.5)}{[0.029, .11] [0.63, 2.32] (0.499) (31.96)}$ |
| CR | 20,000 | 0.72/323 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{13.94 (0) (0.011) (1.077) (0.5)}{[0.088, 0.043] [0.86, 4.86] (0.428) (28.12)}$ |
| CR | 20,000 | 0.88/395 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{19.99 (0) (0.028) (1.341) (0.5)}{(0.069) (-0.045) [0.93, 6.82] (0.415) (24.68)}$ |
| PA | Sea Level | 0.14/95 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{4.26 (0) (0.186) (0.461) (0.5)}{[0.048, 0.26] [0.71, 1.46] (0.5) (32.63)}$ |
| | | | $\frac{\dot{\theta}(1)}{\delta_{ep}} = \frac{4.26 (0.578) (0.5)}{[0.70, 1.47] (0.481) (32.63)}$ |

Note: (1) Phugoid contributions ignored.

The S-3 airplane's pitch rate to cockpit control position command may be represented by a third order numerator over fifth order denominator transfer function of the form:

$$\frac{\dot{\theta}(s)}{\delta_{ep}(s)} = \frac{(0.0052) N_{\dot{\theta}}(s)}{\Delta(s)} \cdot \frac{1}{0.0294s + 1} \quad (a-2)$$

The transfer functions representing the S-3 airplane, as analyzed in this report, are presented in Table A-II.

F-14 — The F-14 airplane is a twin turbo-fan powered, land and carrier based, supersonic fighter aircraft. Longitudinal control is accomplished via an irreversible mechanical flight control system which transmits cockpit control commands to an all moving horizontal stabilizer. Fore and aft bobweights, not modelled in this analysis, are utilized to provide tailored stick force per g characteristics. Pilot force feel is provided via a nonlinear feel spring and stick motions are damped by a sprashpot. The airplane's basic stability is augmented through the feedback of washed out pitch rate which is fed through a shaping network to obtain the desired response. A block diagram of the F-14's longitudinal control system, as modelled in this analysis, is presented in figure A-3. The F-14 airplane's response to longitudinal control inputs may be represented by the following general transfer function:

$$\frac{X(s)}{\delta_{ep}(s)} = \frac{(0.0374) N_X (2s + 1) (0.53s + 1) (0.0715s + 1)}{\Delta(0.05s + 1) (2s + 1) (0.53s + 1) (0.0715s + 1) + 2K_{qs} (0.2s + 1)^2 N_{\dot{\theta}}} \quad (a-3)$$

The transfer functions representing the F-14's response to cockpit control position inputs, as analyzed in this report, are presented in table A-III.

TABLE A-II
S-3 AIRPLANE TRANSFER FUNCTIONS

| Config- uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|--------------------|------------------|----------------------|---|
| CR | 15,000 | 0.36/179 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{249.2 (0) (0.0227) (0.714)}{[0.048, .106] [0.44, 2.59] (34.01)}$ |
| CR | 15,000 | 0.71/353 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{786.7 (0) (0.032) (1.766)}{[0.8, .019] [0.48, 5.45] (34.01)}$ |
| PA | Sea Level | 0.15/97 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{73.43 (0) (0.173) (0.594)}{[0.18, 0.197] [0.57, 1.53] (34.01)}$ $\frac{\dot{\theta}(1)}{\delta_{ep}} = \frac{73.43 (0.675)}{[0.57, 1.52] (34.01)}$ |

Note: (1) Phugoid contributions ignored.

TABLE A-III
F-14 AIRPLANE TRANSFER FUNCTIONS

| Config- uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|--------------------|------------------|----------------------|---|
| CR | 15,000 | 0.5/248 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{5.26 (0) (0.0103) (0.773) (0.5) (1.887) (13.986)}{[.016, .082] [.61, 2.78] (.418) (1.34) [.97, 17.04]}$ $\frac{N_{zcg}}{\delta_{ep}} = \frac{-1.50 (0) (0.00066) (6.619) (-6.728) (0.5) (1.887) (13.986)}{D(1)}$ $\frac{N_{zcr}}{\delta_{ep}} = \frac{1.34 (0) (0.00066) (49.99) (0.5) (1.887) (13.986)}{D}$ |
| CR | 15,000 | 0.7/348 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{11.48 (0) (0.0119) (0.962) (0.5) (1.887) (13.986)}{[0.057, .065] [0.58, 4.27] (0.408) (1.532) [0.91, 17.51]}$ $\frac{N_{zcg}}{\delta_{ep}} = \frac{-3.22 (0) (0.0079) (8.816) (-8.919) (0.5) (1.887) (13.986)}{D}$ $\frac{N_{zcr}}{\delta_{ep}} = \frac{3.45 (0) (0.0079) (73.55) (0.5) (1.887) (13.986)}{D}$ |

Note: (1) D = Denominator of $\dot{\theta}/\delta_{ep}$ transfer function.

TABLE A-III
F-14 AIRPLANE TRANSFER FUNCTIONS
(Continued)

| Config- uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|----------------------|------------------|----------------------|---|
| CR | 15,000 | 0.83/412 | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{15.22 (0) (0.012) (0.965) (0.5) (1.887) (13.986)}{[0.11, 0.058] [0.62, 5.01] (0.418) (1.562) [0.87, 17.95]}$ |
| | | | $\frac{N_{zcg}}{\delta_{ep}} = \frac{-4.16 (0) (0.0093) (9.78) (-9.856) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{N_{zcr}}{\delta_{ep}} = \frac{4.35 (0) (0.0093) (92.38) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{30.46 (0) (0.0295) (1.143) (0.5) (1.887) (13.986)}{[0.25, 0.064] [0.71, 6.88] (0.418) (1.685) [0.70, 20.37]}$ |
| CR | 15,000 | 1.2/596 | $\frac{N_{zcg}}{\delta_{ep}} = \frac{-7.04 (0) (0.0278) (13.75) (-14.17) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{N_{zcr}}{\delta_{ep}} = \frac{11.02 (0) (0.0278) (124.4) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{1.179 (0.473) (0.5) (1.887) (13.986)}{[0.52, 1.48] (0.506) (1.591) (15.09) (18.66)}$ |
| | | | $\frac{N_{zcg}}{\delta_{ep}} = \frac{-5.587 (3.185) (-2.735) (0.5) (1.887) (13.986)}{D^{(1)}}$ |
| PA(1) ⁽²⁾ | Sea Level | .19/126 | $\frac{N_{zcr}}{\delta_{ep}} = \frac{0.134 (363.6) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{\dot{\theta}}{\delta_{ep}} = \frac{1.034 (0.444) (0.5) (1.887) (13.986)}{(0.7, 1.05) (0.531) (1.48) (14.91) (18.87)}$ |
| | | | $\frac{N_{zcg}}{\delta_{ep}} = \frac{-5.899 (-2.357) (2.756) (0.5) (1.887) (13.986)}{D}$ |
| | | | $\frac{N_{zcr}}{\delta_{ep}} = \frac{0.264 (145.49) (0.5) (1.887) (13.986)}{D}$ |
| PA(2) ⁽²⁾ | Sea Level | .18/121 | |
| | | | |
| | | | |
| | | | |

Notes: (1) D = Denominator of $\dot{\theta}/\delta_{ep}$ transfer function.

(2) Phugoid contributions ignored.

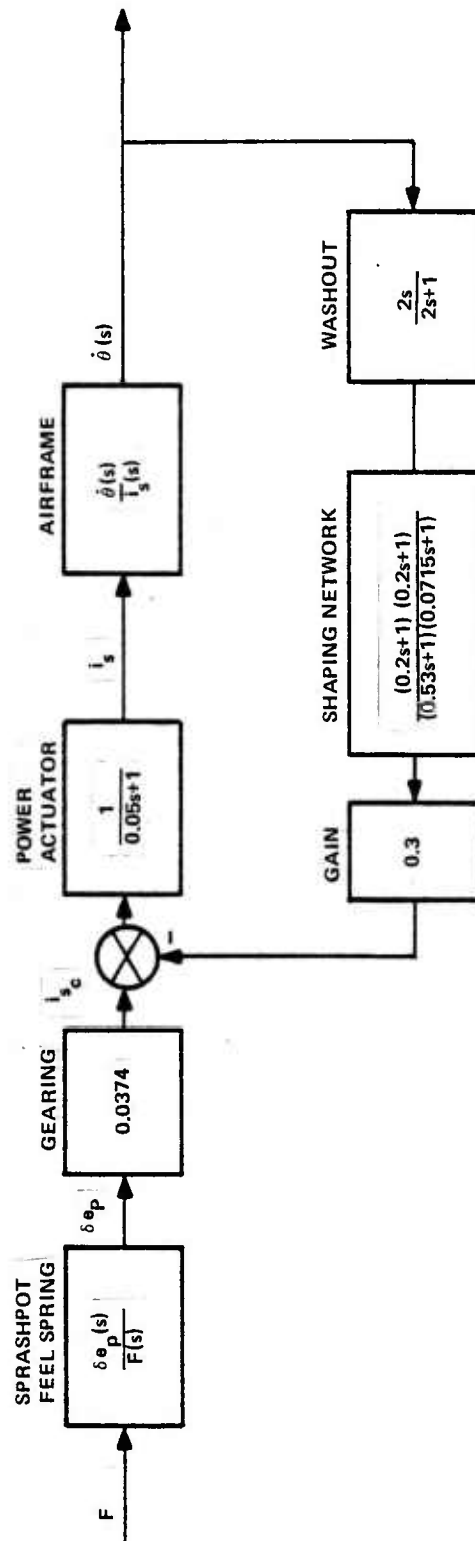


Figure A-3 — F-14 Airplane Longitudinal Control System Block Diagram

The response to cockpit force commands is obtained from:

$$\frac{X(s)}{F_x(s)} = \frac{X(s)}{\delta_{ep}(s)} \cdot \frac{\delta_{ep}(s)}{F_x(s)} \quad (a-4)$$

where:

$$\frac{\delta_{ep}(s)}{F_x(s)} = \frac{26.825 (s + 39.815)}{(s + 3.366) (s^2 + 36.45s + 1580)} , \delta_{ep} \leq 1 \text{ in} \quad (a-5)$$

A-7 — The A-7 airplane is a single place turbo fan powered, land and carrier based, light attack aircraft. It contains an irreversible mechanical longitudinal control system with both stability and control augmentation. The stability augmentation system provides pitch rate and filtered normal acceleration feedback signals to augment the aircraft's basic stability characteristics. The command augmentation system feeds control force signals forward through a prefilter as a means of increasing the pilot's commanded input. The A-7 control stick dynamics were not modelled in this analysis. A simplified block diagram of the A-7 airplane's longitudinal control system is presented in figure A-4. The A-7 airplane's response to control force inputs may be represented by the following general transfer function:

$$\frac{X(s)}{F_x(s)} = \frac{N_X [(0.55s + 1) K_m + K_{cas}]}{\Delta (0.55s + 1) (0.05s + 1) + K_{nza} N_{nza} + K_{\dot{\theta}} N_{\dot{\theta}}} \quad (a-6)$$

The transfer functions representing the A-7 airplanes response to pilot force commands, as modelled in this analysis, are presented in table A-IV.

F-18 — The F-18 is a single-place, turbo-powered, land and carrier based, fighter aircraft controlled by a digital flight control system. Separate flight control laws are provided for differing flight regimes. Electrical signals are generated from the pilot's control force inputs, passed to the computer, modified by various gain and shaping networks, and finally passed to the all-moving horizontal stabilizer. Normal acceleration and pitch rate signals are also input to the computer, where they are shaped and gain scheduled before being summed with the command input signals. The cockpit control feed system dynamics were not included in this analysis. Simplified block diagrams of the F-18 airplane's longitudinal flight control system (revision 4.1) are presented in figures A-5 and A-6 for the Cruise and Power Approach flight regimes, respectively. The F-18 high order system transfer functions investigated in this analysis, obtained from the NAVAIRDEVCON Flight Control Section, Code 6012, are presented in table A-V.

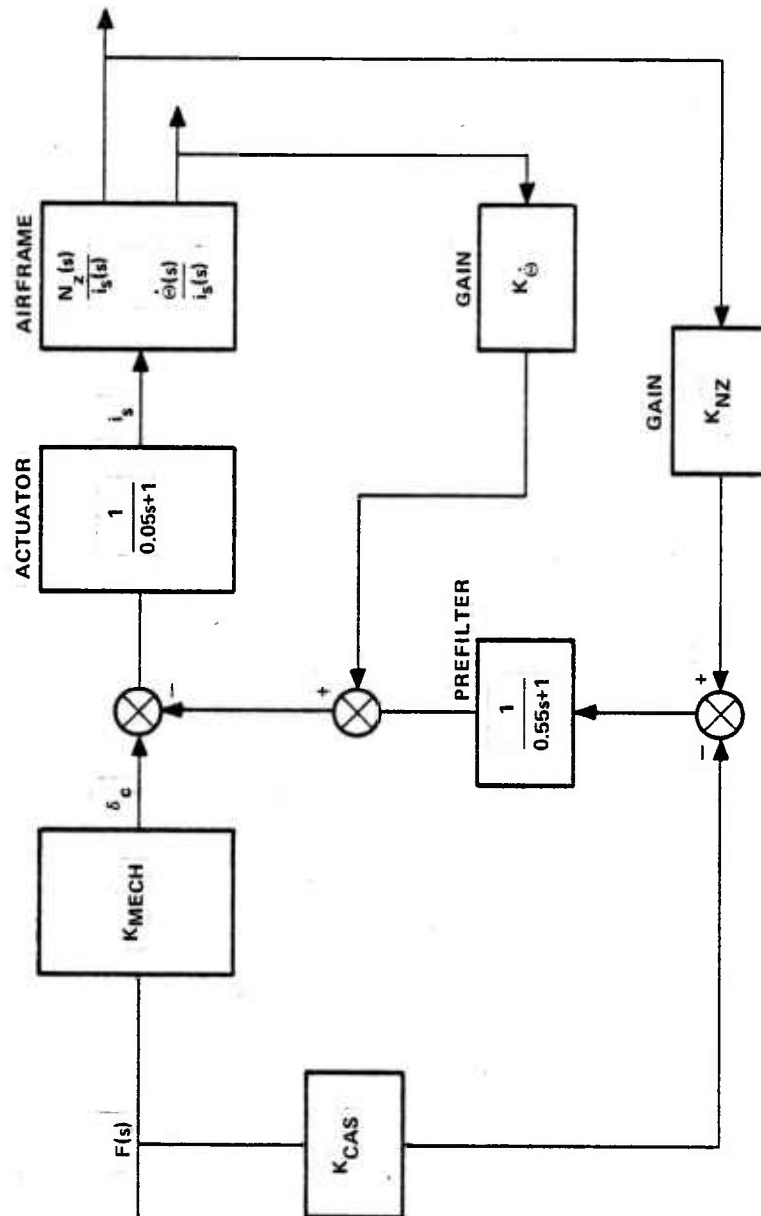


Figure A-4 — A-7 Airplane Longitudinal Control System Block Diagram

TABLE A-IV
A-7 AIRPLANE TRANSFER FUNCTIONS

| Config-uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|----------------|---------------|-------------------|---|
| CR | 15,000 | 0.3/149 | $\frac{\dot{\theta}}{F} = \frac{0.158 (0) (-0.0082) (0.506) (7.272)}{[0.064, 0.12] [0.52, 1.83] (2.079) (18.758)}$ $\frac{n_{z_{cg}}}{F} = \frac{-0.833 (-0.0042) (-0.05) (5.55) (-5.08) (7.272)}{D^{(1)}}$ $\frac{n_{z_{cr}}}{F} = \frac{0.08 (-0.0042) (-0.05) (340) (7.272)}{D}$ |
| CR | 15,000 | 0.6/298 | $\frac{\dot{\theta}}{F} = \frac{0.658 (0) (0.0072) (1.09) (7.272)}{[0.068, .05] [0.76, 3.23] (7.069) (11.84)}$ |
| CR | 15,000 | 0.9/447 | $\frac{\dot{\theta}}{F} = \frac{1.456 (0) (0.0443) (1.97) (7.272)}{(0.0644) (-0.0255) [0.78, 4.01] [0.64, 14.94]}$ |

Note: (1) D = Denominator of $\dot{\theta}/\delta_{ep}$ Transfer Function

TABLE A-V
F-18 AIRPLANE TRANSFER FUNCTIONS

| Config-uration | Altitude (ft) | Airspeed (M/KEAS) | Transfer Function |
|----------------|---------------|-------------------|--|
| CR | 10,000 | 0.5/274 | $\frac{\dot{\theta}}{F} = \frac{6284.5(0)(0.25)(1)(1)(1.247)(1.25)(3.33)(30.03)[0.03,60]}{(-0.0026)(0.22)[0.81,1.11](0.89)(1.67)[0.76,4.47][0.76,30][0.35,55][-0.80,61.7]}$ |
| PA | Sea Level | .20/133 | $\frac{\dot{\theta}}{F} = \frac{170.7(0.395)(1)(1)(14.13)(20.32)(25.08)[0.03,60]}{[0.682,1.78](0.368)(2)(15.82)(19.82)(24.71)[0.88,50][0.66,65.1]}$ $\frac{n_{z_{cr}}}{F} = \frac{2.45(1)(1)(8.85)[0.88,22.7](32.06)[0.03,60]}{D^{(1)}}$ |

Note: (1) D = Denominator of θ/δ_{ep} Transfer Function

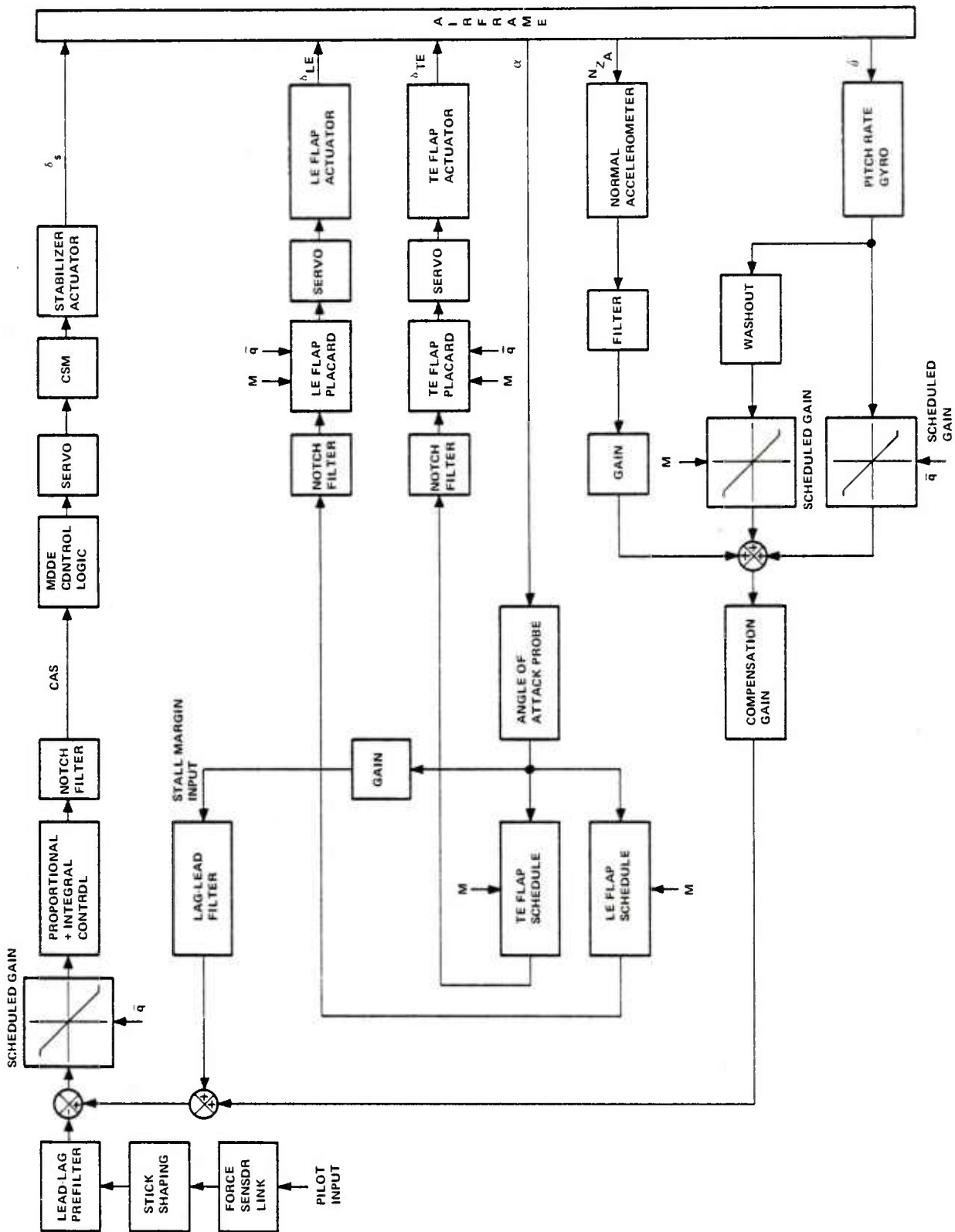


Figure A-5 -- F-18 Airplane Longitudinal Control System Block Diagram -- Cruise Configuration

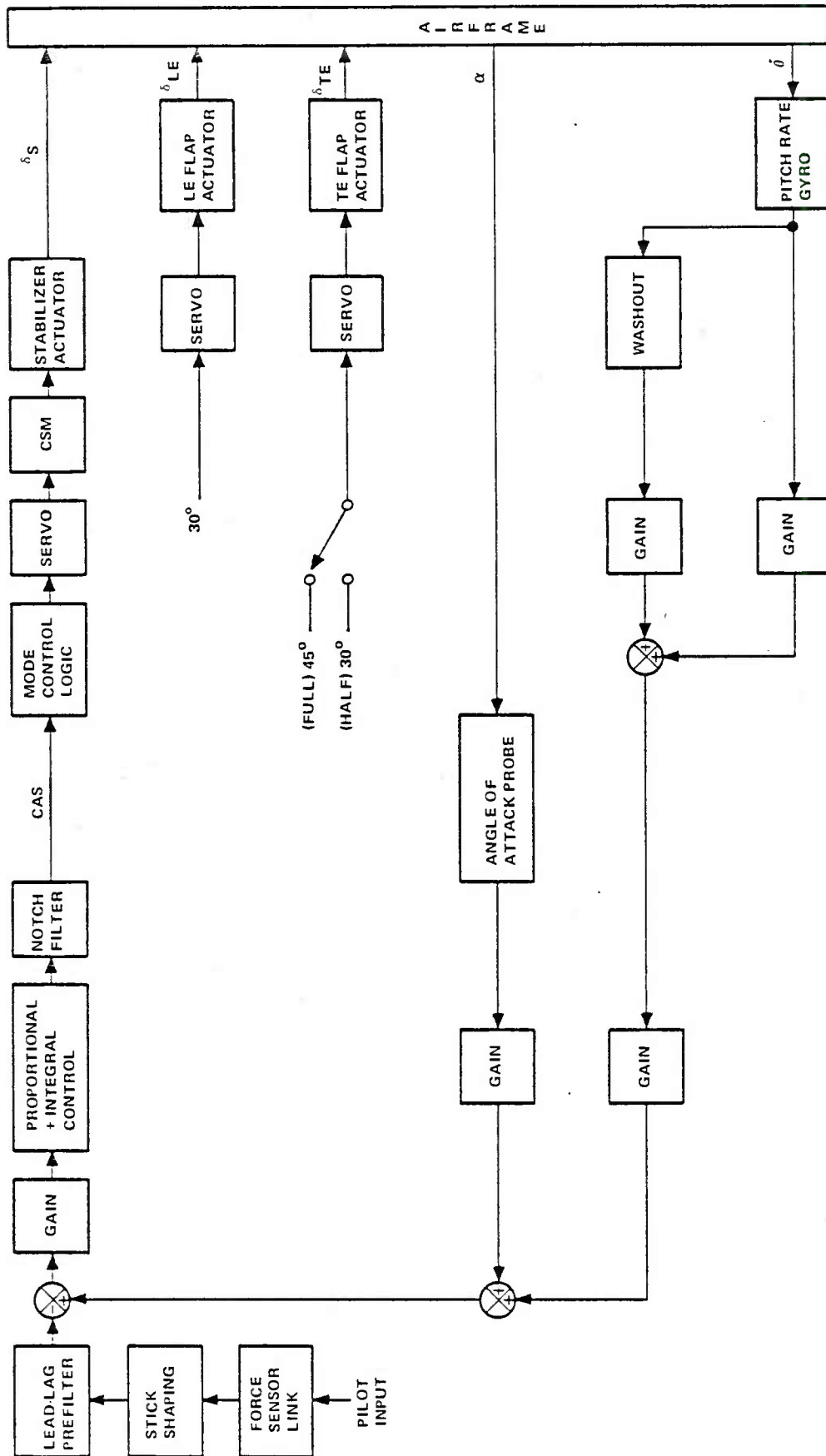


Figure A-6 — F-18 Airplane Longitudinal Control System Block Diagram — Power Approach Configuration

APPENDIX B

Frequency and Time History Response Comparisons

Bode plots showing frequency response and time history plots showing the airplane's calculated response to step control inputs are compared for the assumed high order and various low order systems in the illustrations of this appendix, figures B-1 through B-24. Data are presented for each of the airplanes analyzed at one flight condition in both cruise and power approach configurations.

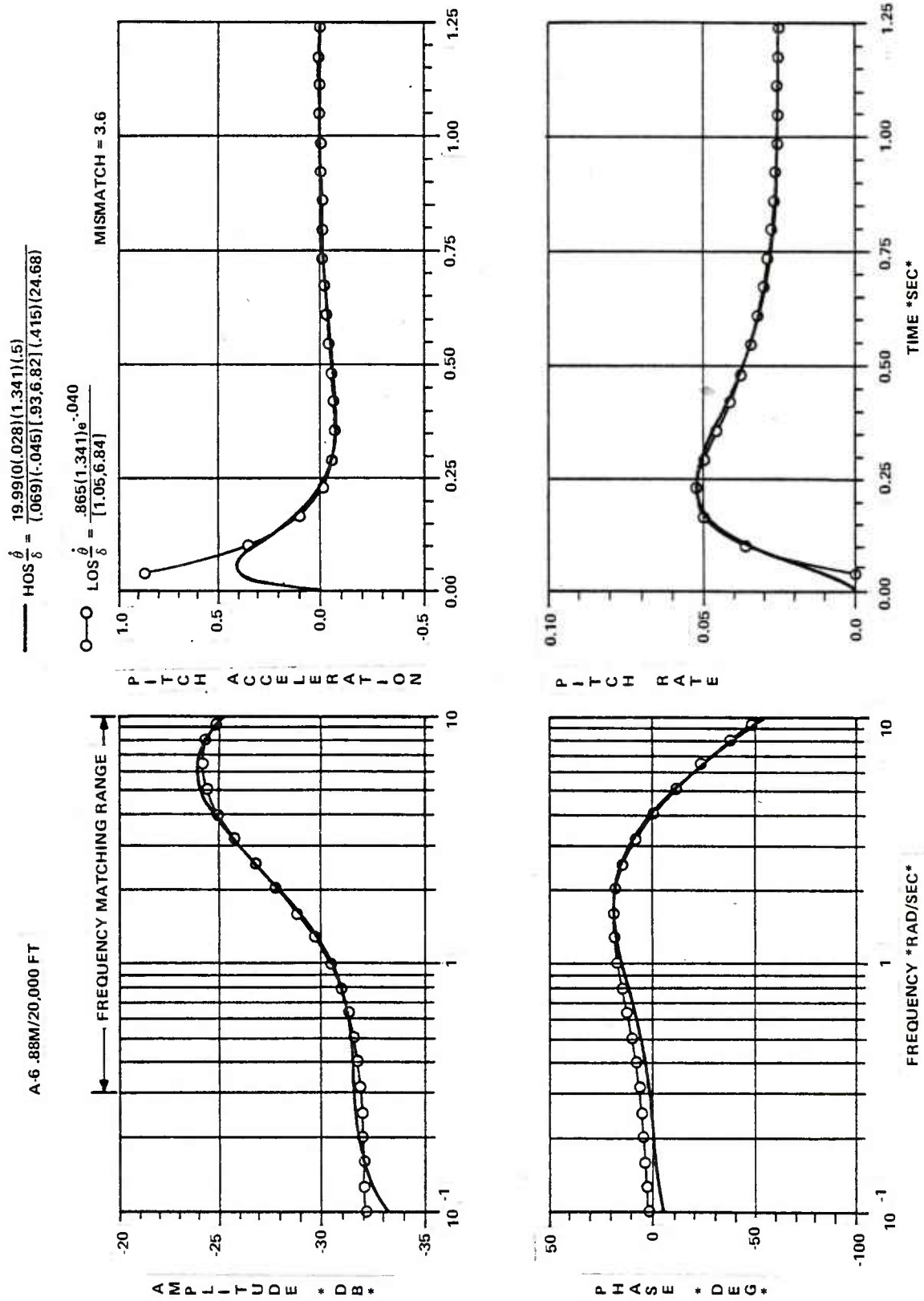
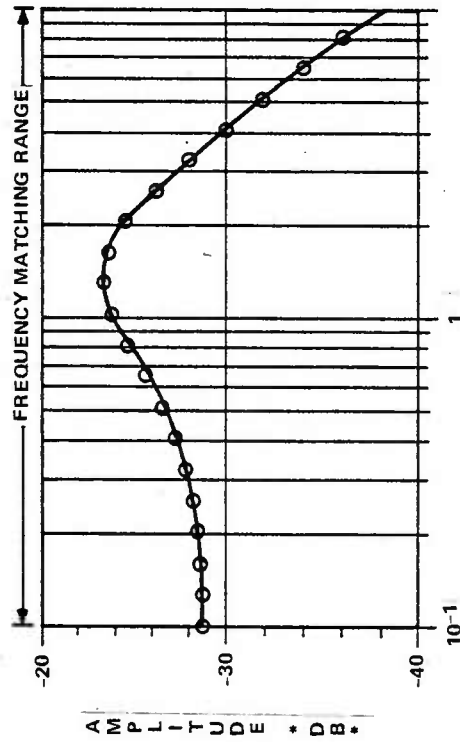


Figure B-1 — A-6 Airplane Pitch Rate Comparisons - Configuration CR - L_{α} Fixed

A-6 95 KEAS



$$\text{HOS } \frac{\dot{\theta}}{\delta} = \frac{4.26(.578)(.5)}{[.70, .481](.481)(32.63)}$$

$$\text{LOS } \frac{\dot{\theta}}{\delta} = \frac{.129(.578)e^{-.030}}{[.71, 1.44]}$$

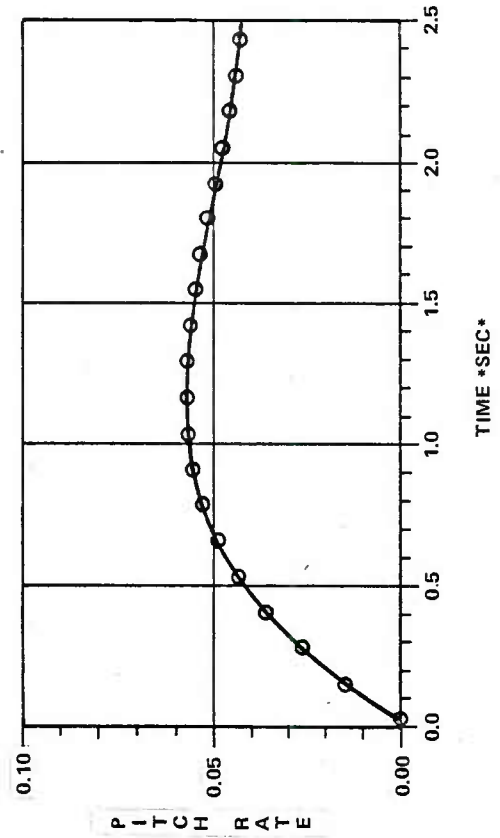
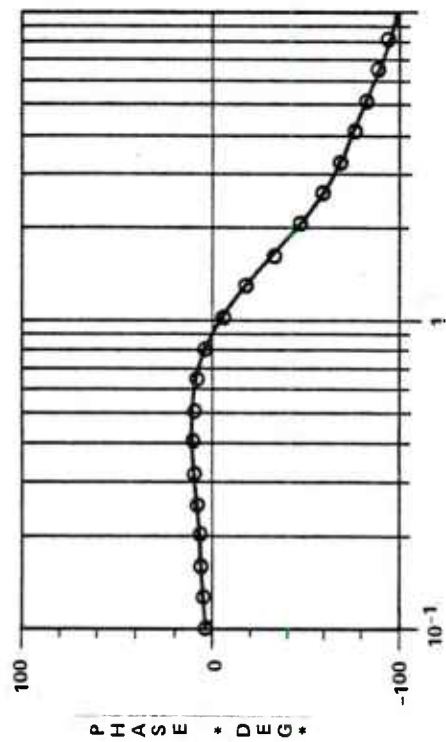
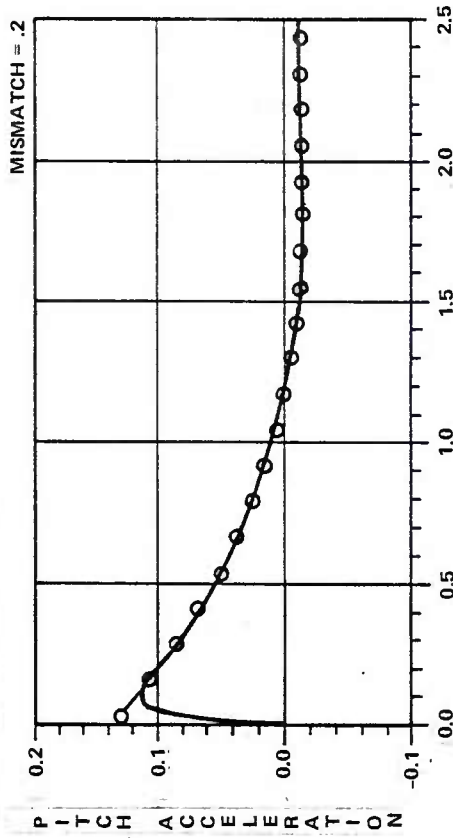


Figure B-2 — A-6 Airplane Pitch Rate Comparisons - Configuration PA - L_{α} Fixed

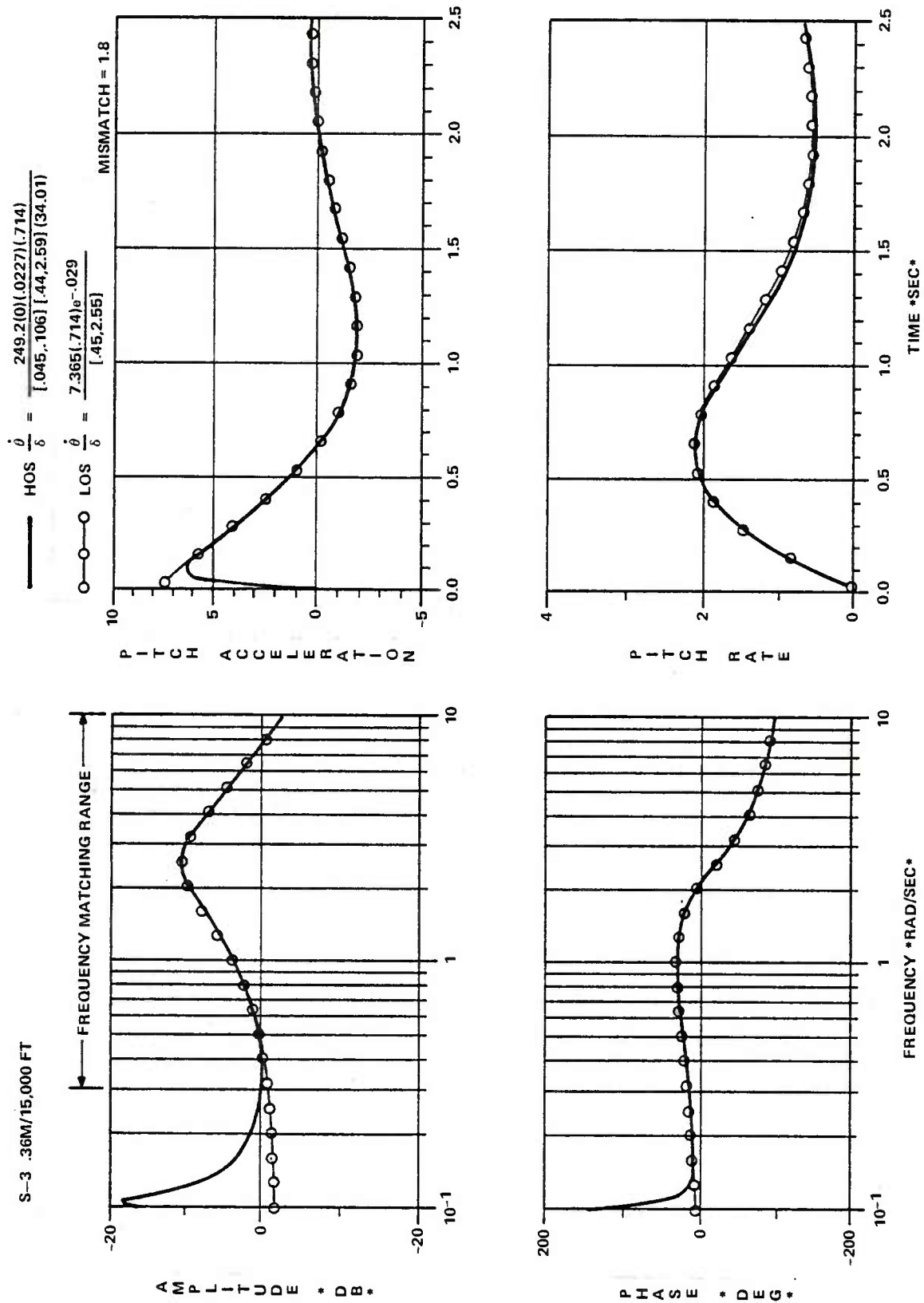


Figure B-3 — S-3 Airplane Pitch Rate Comparisons Configuration CR - L_α Fixed

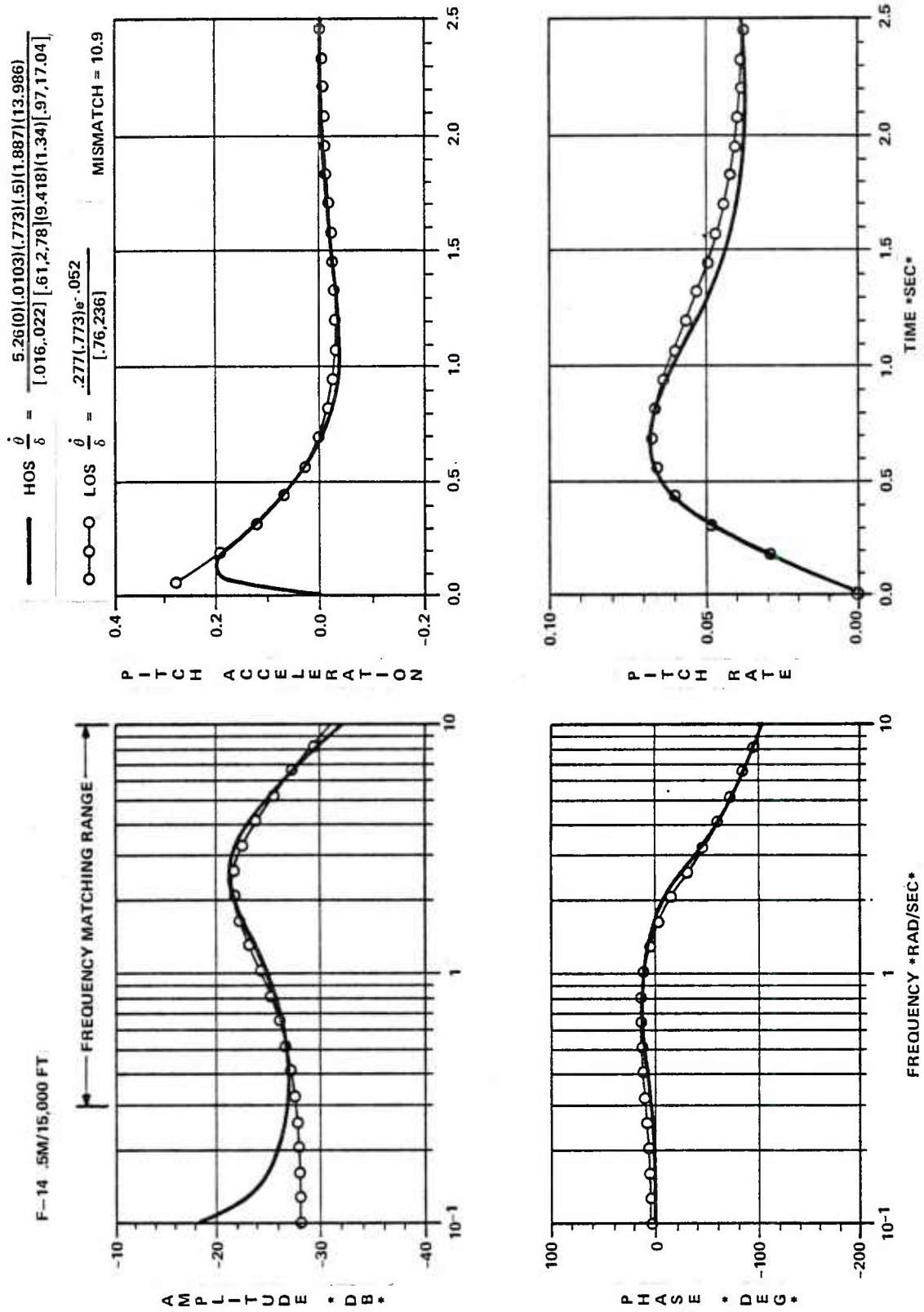


Figure B-4 — F-14 Airplane Pitch Response to Control Position Inputs - Configuration CR - L_{α} Fixed

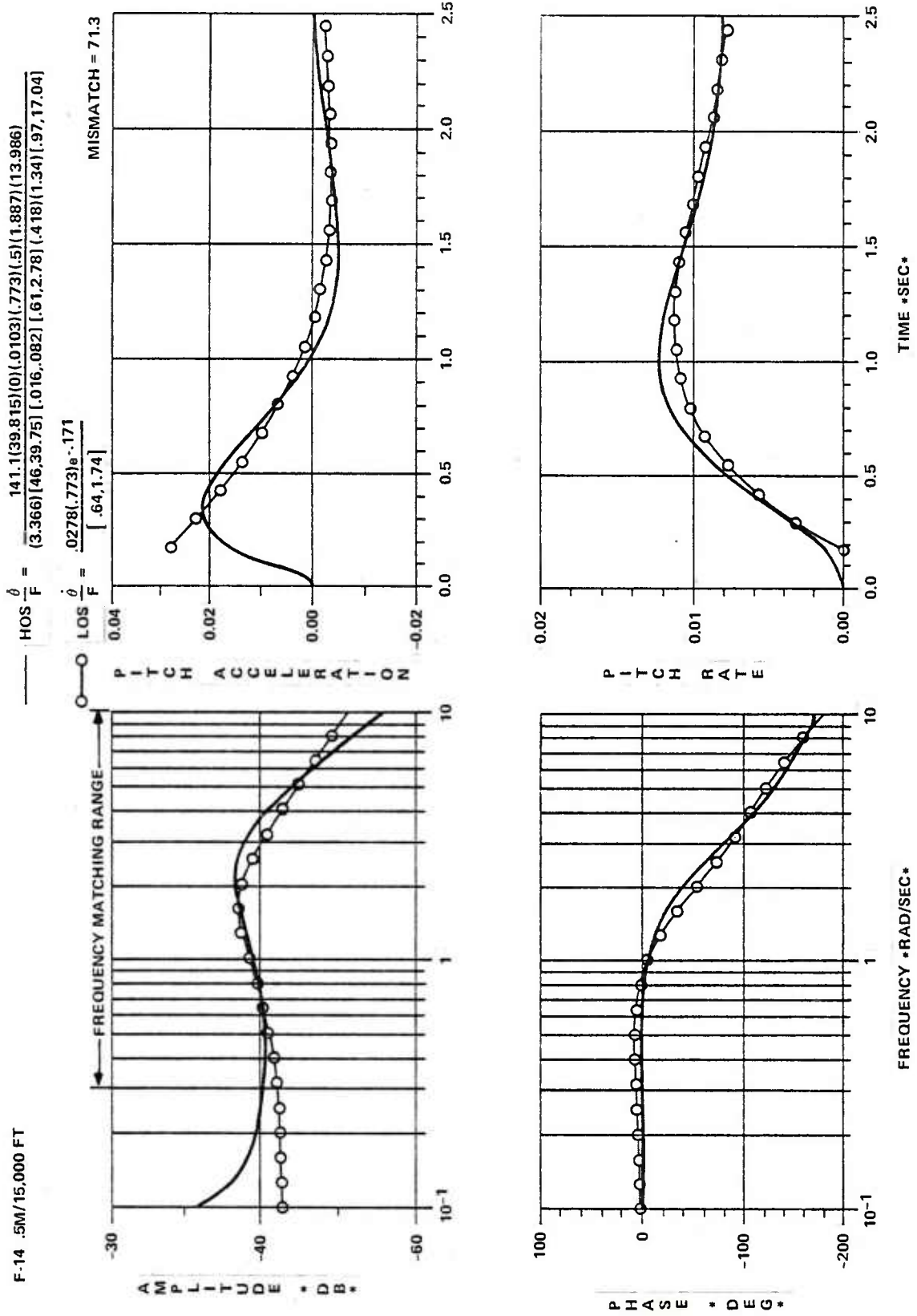


Figure B-5 — F-14 Airplane Pitch Response to Control Force Inputs-Configuration CR - L_{α} Fixed

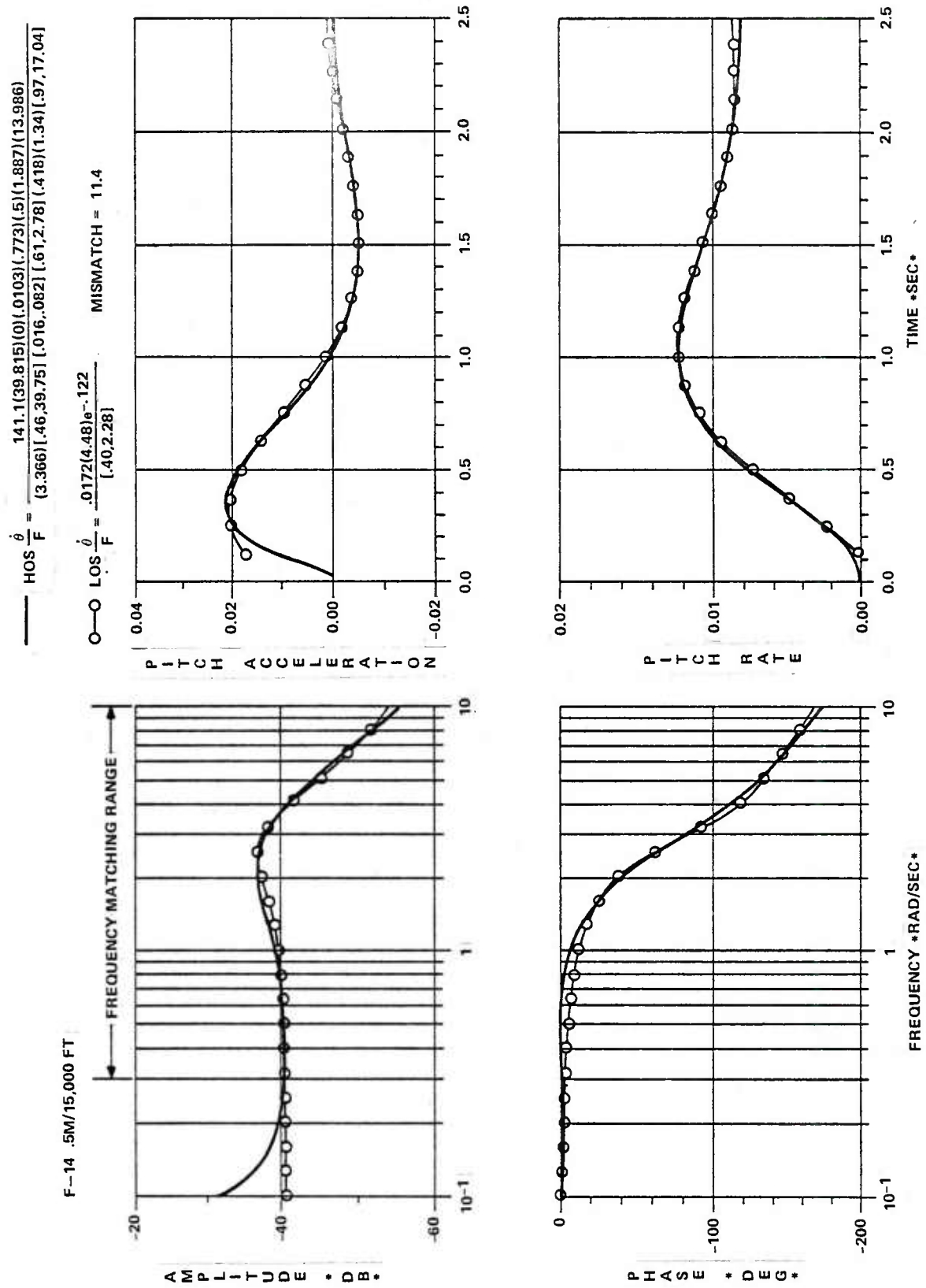


Figure B-6 — F-14 Airplane Pitch Response to Control Force Inputs - Configuration CR - L_α Free

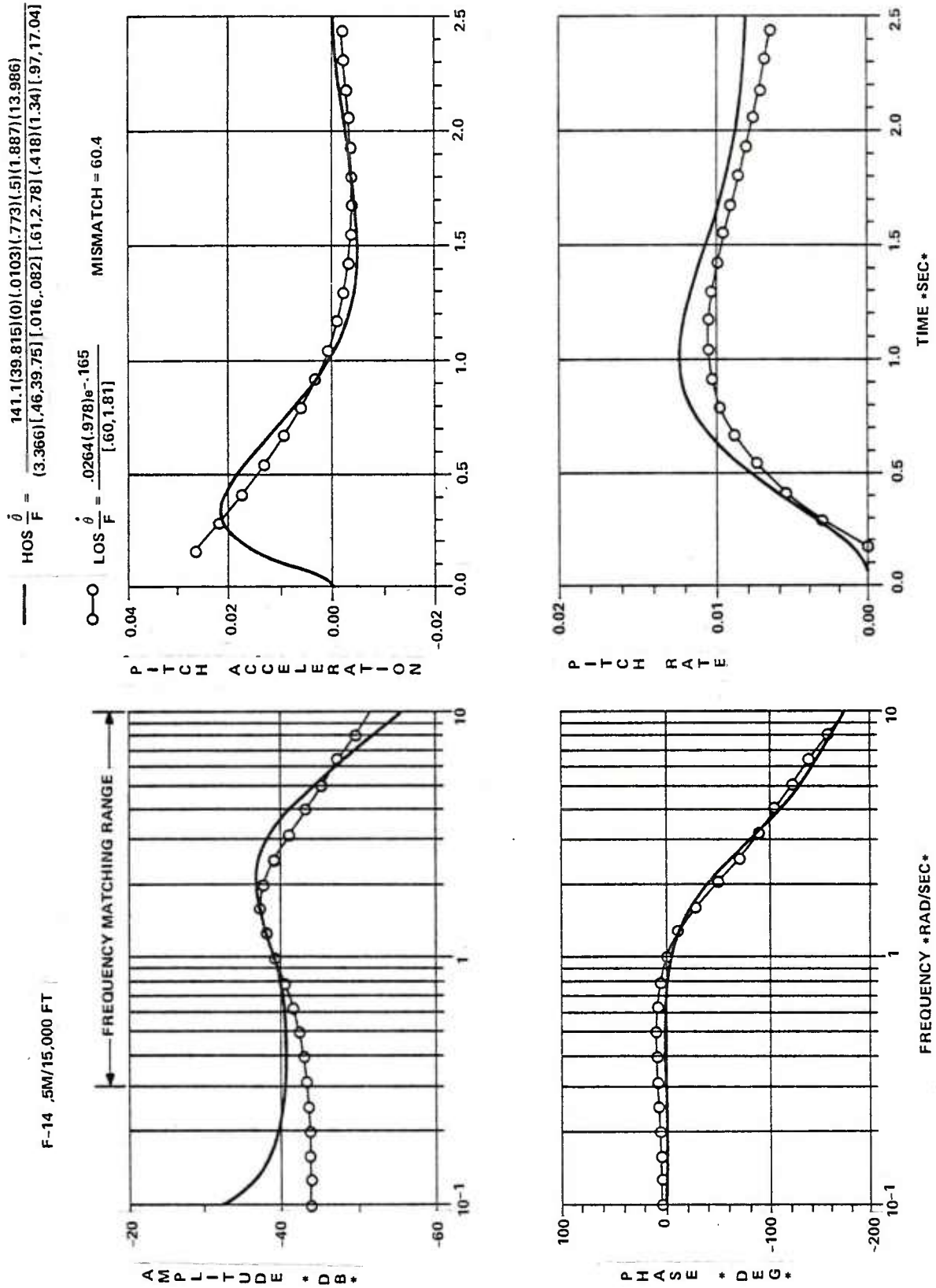
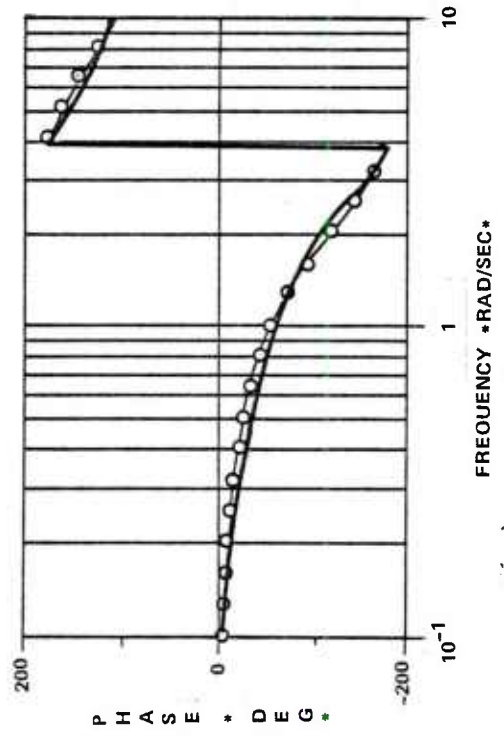
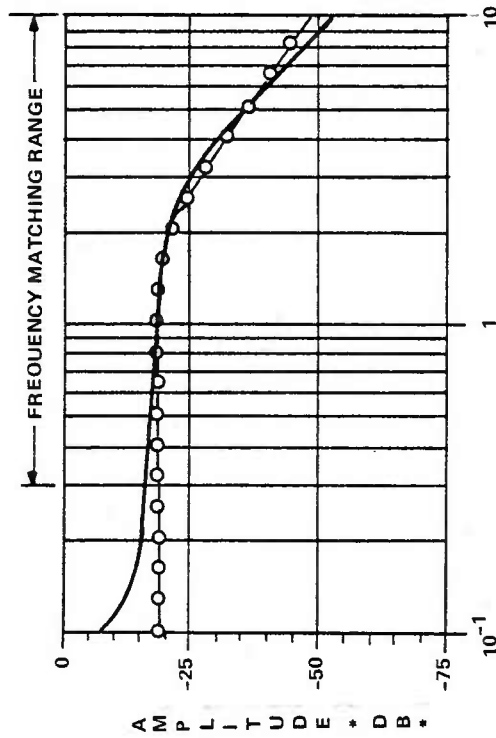


Figure B-7 — F-14 Airplane Pitch Response to Control Force Inputs - Configuration CR - Simultaneous $\dot{\theta}$ and Nz Matching

F-14 .5M/15,000 FT



$$\text{HOS } \frac{NZ}{F} = \frac{35.946(0)[.00066](49.99)[.5](1.887)(13.986)(39.815)}{[.016, .082][.61, 2.78][.418](1.34)(3.366)[.97, 17][.46, 39.75]}$$

$$\text{O} \text{---} \text{O } \text{LOS } \frac{NZ}{F} = \frac{.364e^{-.153}}{[.60, 1.81]} \quad \text{MISMATCH} = 71.3$$

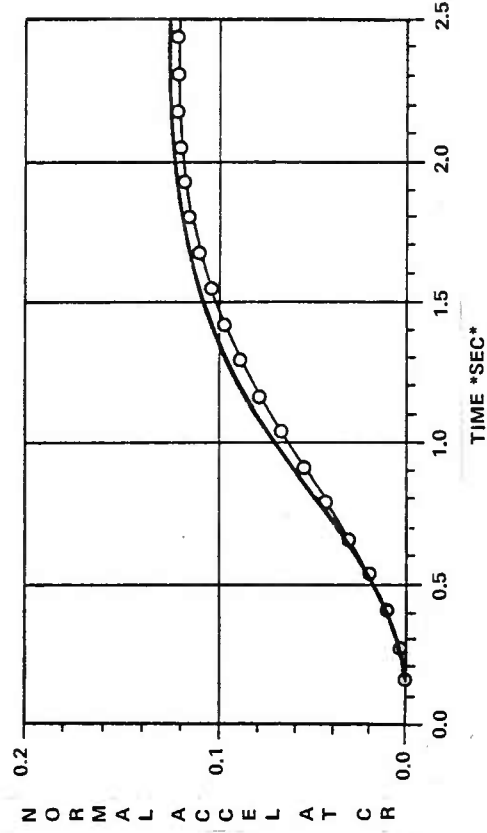
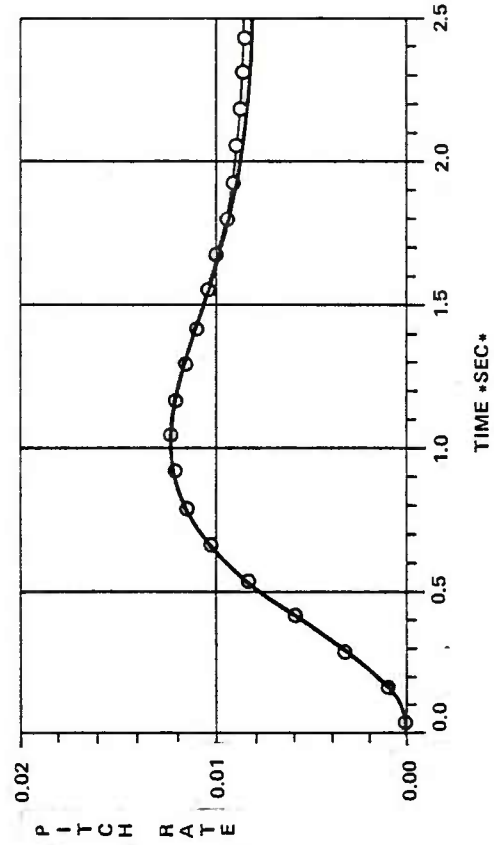
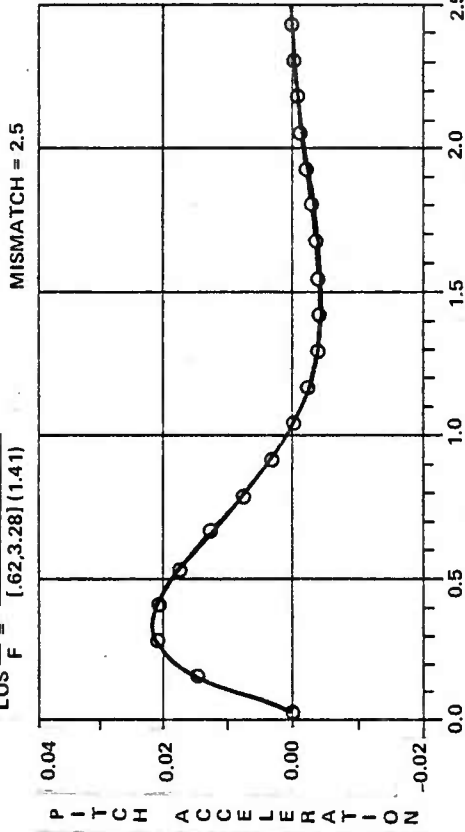


Figure B-8 — F-14 Airplane Normal Acceleration Response to Control Force Inputs - Configuration CR - Simultaneous θ and Nz Matching

$$\text{HOS } \frac{\delta}{F} = \frac{141.1(39.815)(0)(.0103)(.773)(.5)(1.887)(13.986)}{(3.366)(.46,39.75)(.016,.082)(.61,2.78)(.418)(1.34)(.97,17.04)}$$

$$\text{LOS } \frac{\delta}{F} = \frac{.159(.773)e^{-.036}}{(.62,3.28)(1.41)}$$



F-14 .5M/15,000 FT

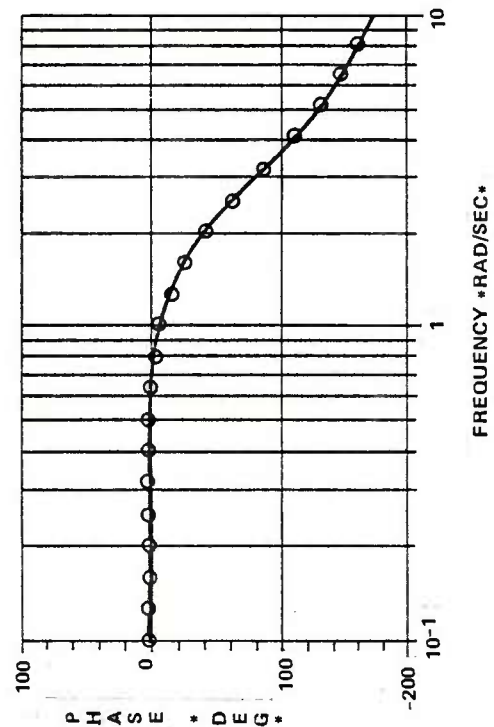
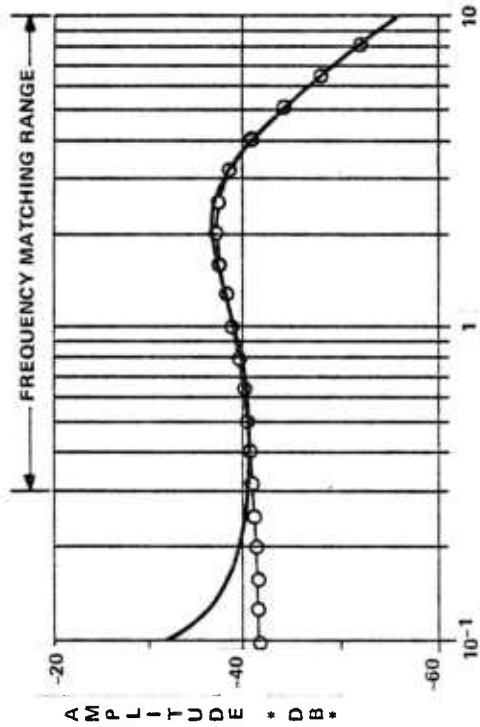


Figure B-9 — F-14 Airplane First Over Third Order Pitch Response to Control Force Inputs - Configuration CR - L_{α} Fixed

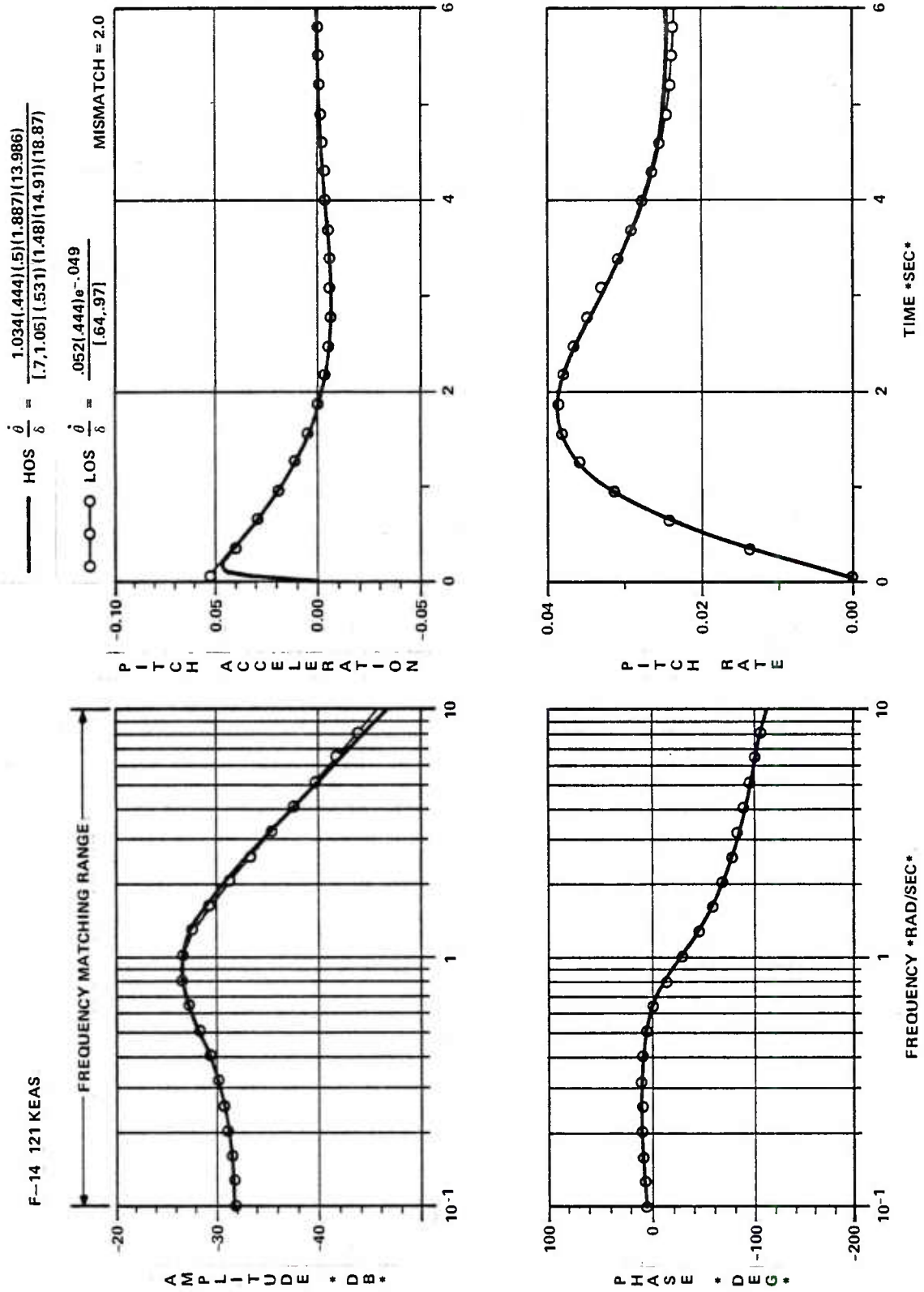


Figure B-10 — F-14 Airplane Pitch Response to Control Position Inputs - Configuration PA(2) - L_α Fixed

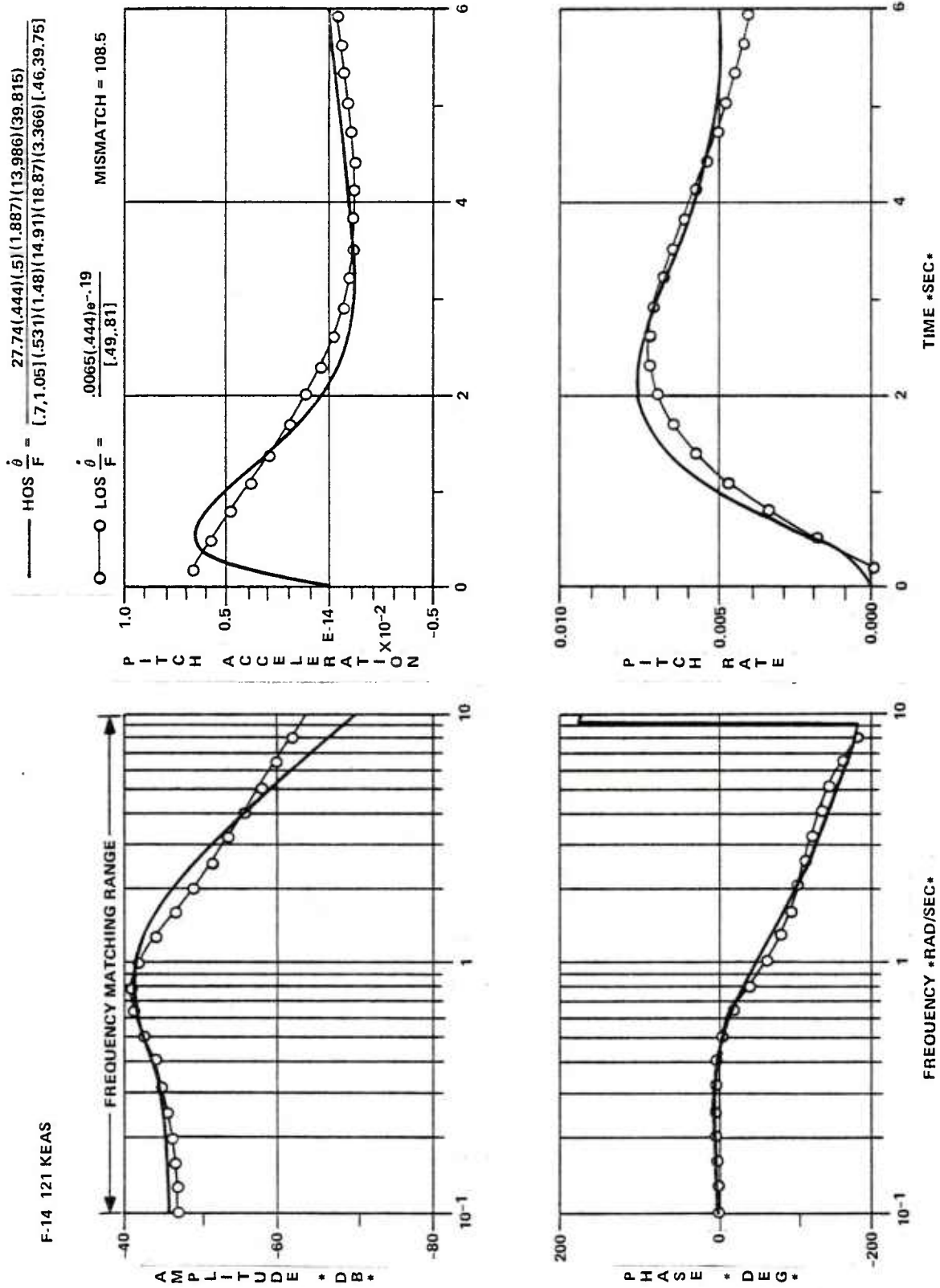


Figure B-11 — F-14 Airplane Pitch Response to Control Force Inputs - Configuration PA(2) - L_{α} Fixed

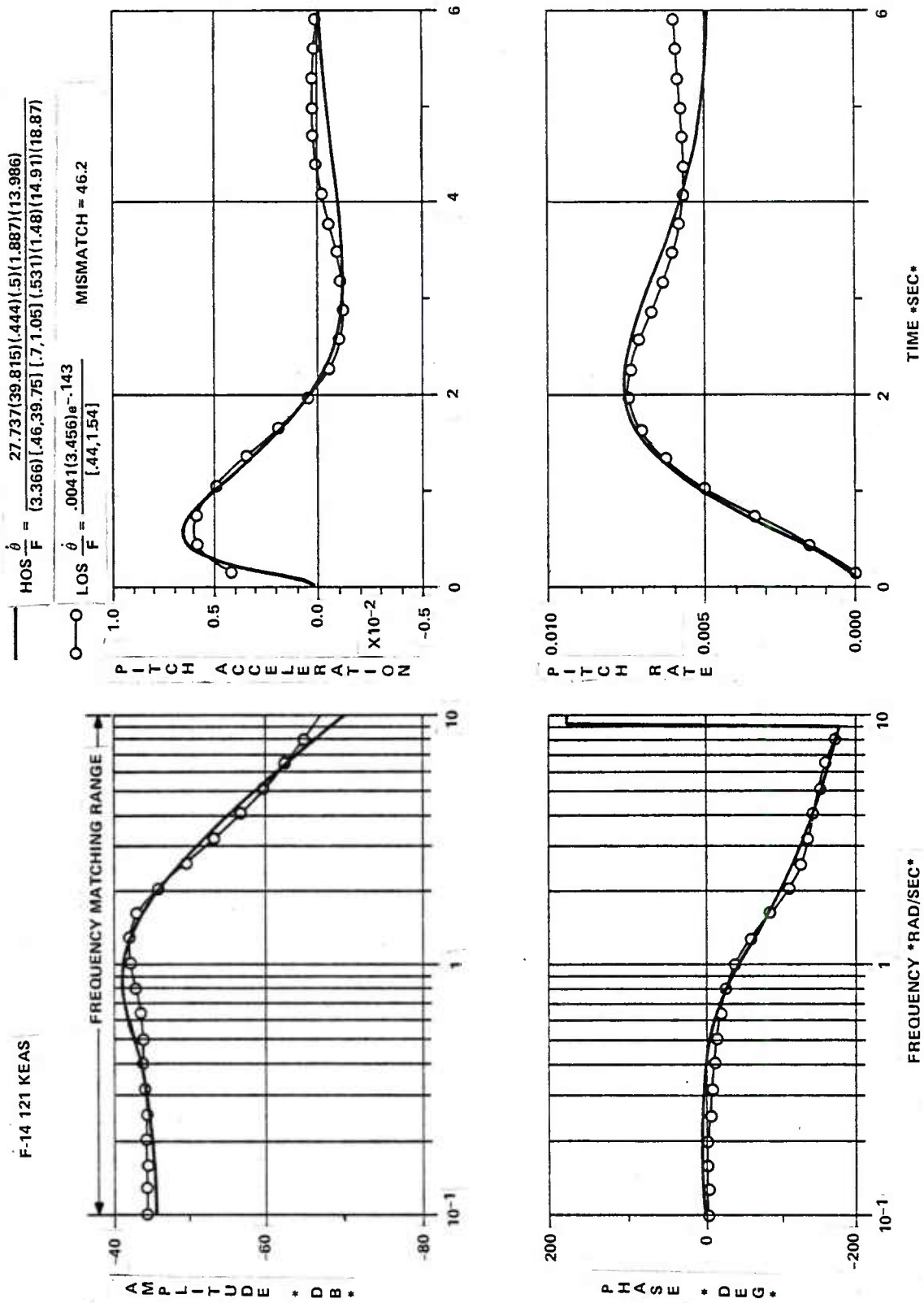


Figure B-12 — F-14 Airplane Pitch Response to Control Force Inputs - Configuration PA(2) - L_α Free

$$\text{HOS } \frac{\dot{\theta}}{F} = \frac{27.737(39.815)(.444)(.5)(1.887)(13.986)}{(3.366)[.46, 39.75][.76, 1.05](.531)(1.48)(14.91)(18.87)}$$

$$\text{LOS } \frac{\dot{\theta}}{F} = \frac{.0062(.545)e^{-.186}}{[.47, .84]} \quad \text{MISMATCH} = 103.9$$

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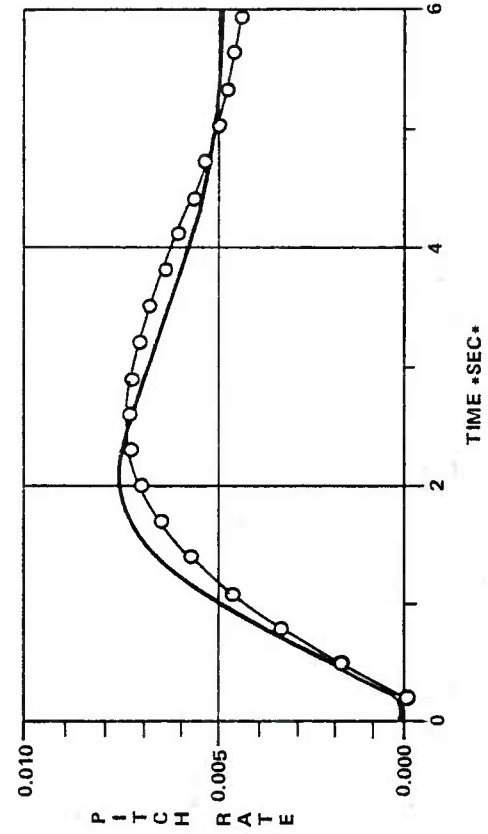
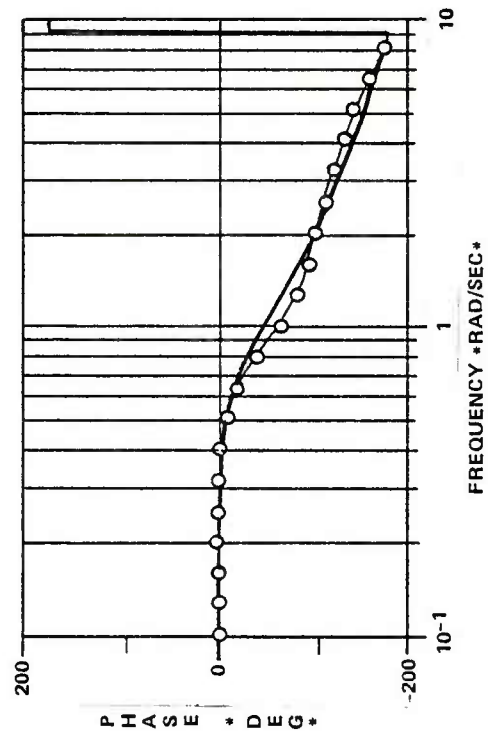
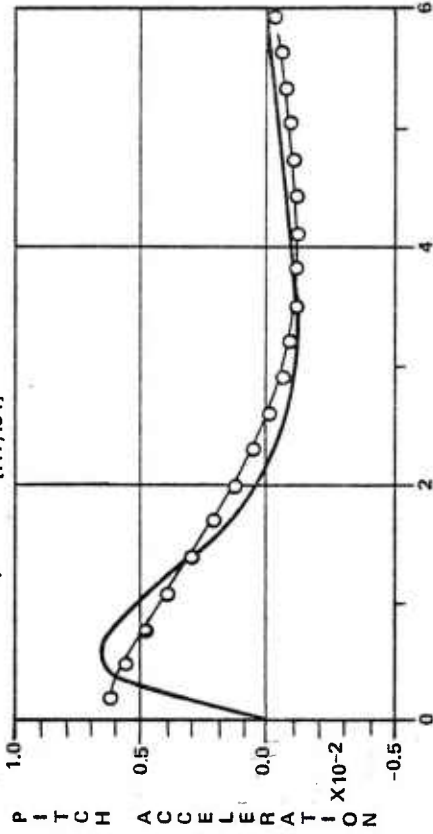
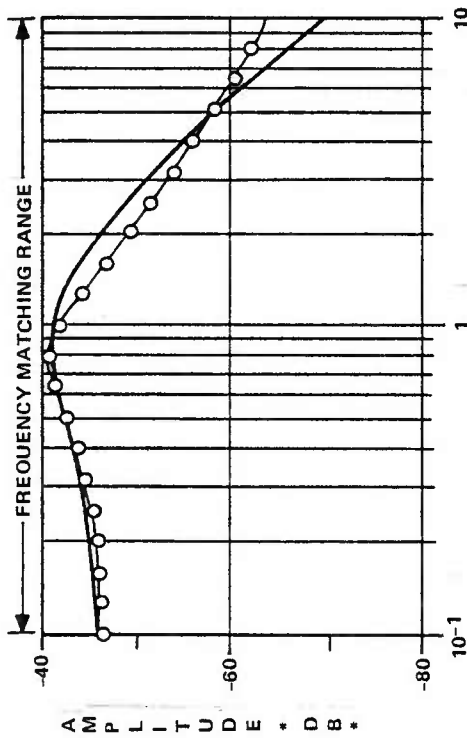
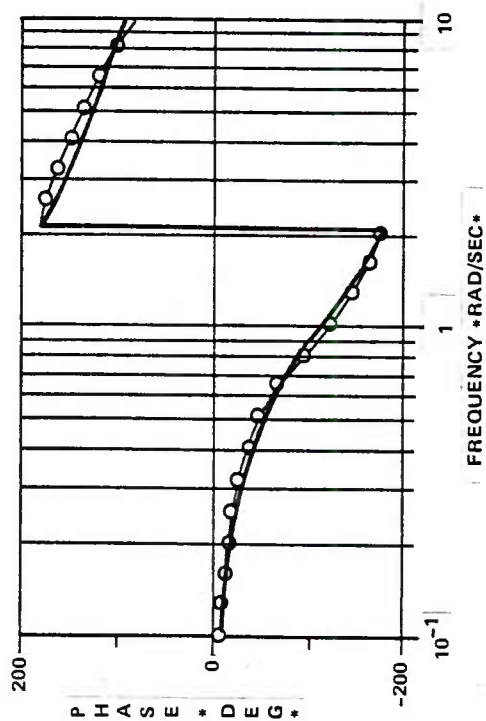
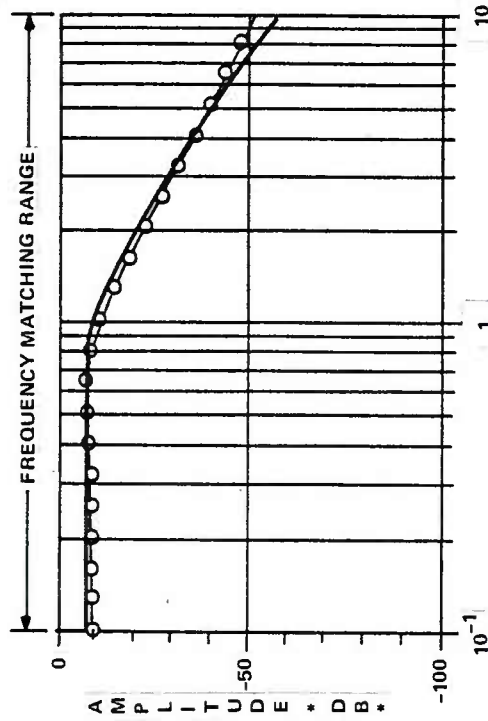


Figure B-13 — F-14 Airplane Pitch Response to Control Force Inputs - Configuration PA(2) - Simultaneous $\dot{\theta}$ and N_z Matching

F-14 121 KEAS



$$\text{HOS} \frac{NZ}{F} = \frac{7.082(145.5)(.5)(1.887)(13.986)(39.815)}{[.7, 1.05](.53)(1.48)(14.91)(18.87)(3.366)[.46, 39.75]}$$

$$\text{LOS} \frac{NZ}{F} = \frac{.246e^{-.184}}{[.47, .84]} \quad \text{MISMATCH} = 116.7$$

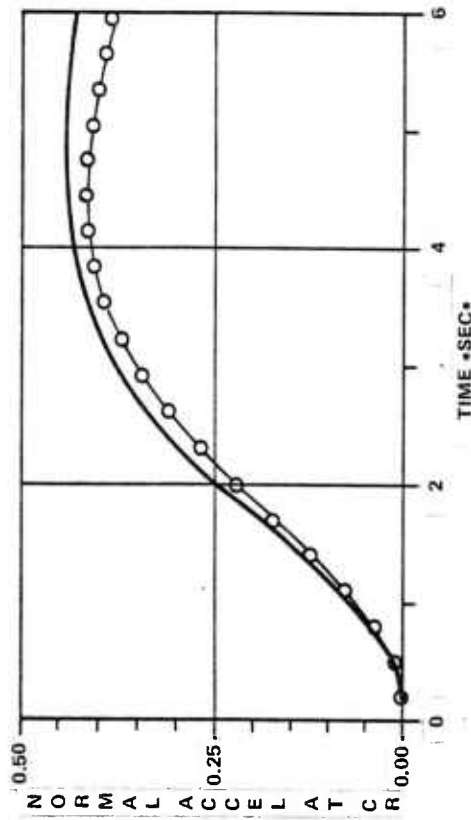


Figure B-14 — F-14 Airplane Normal Acceleration Response to Control Force Inputs - Configuration PA(2) - Simultaneous θ and Nz Matching

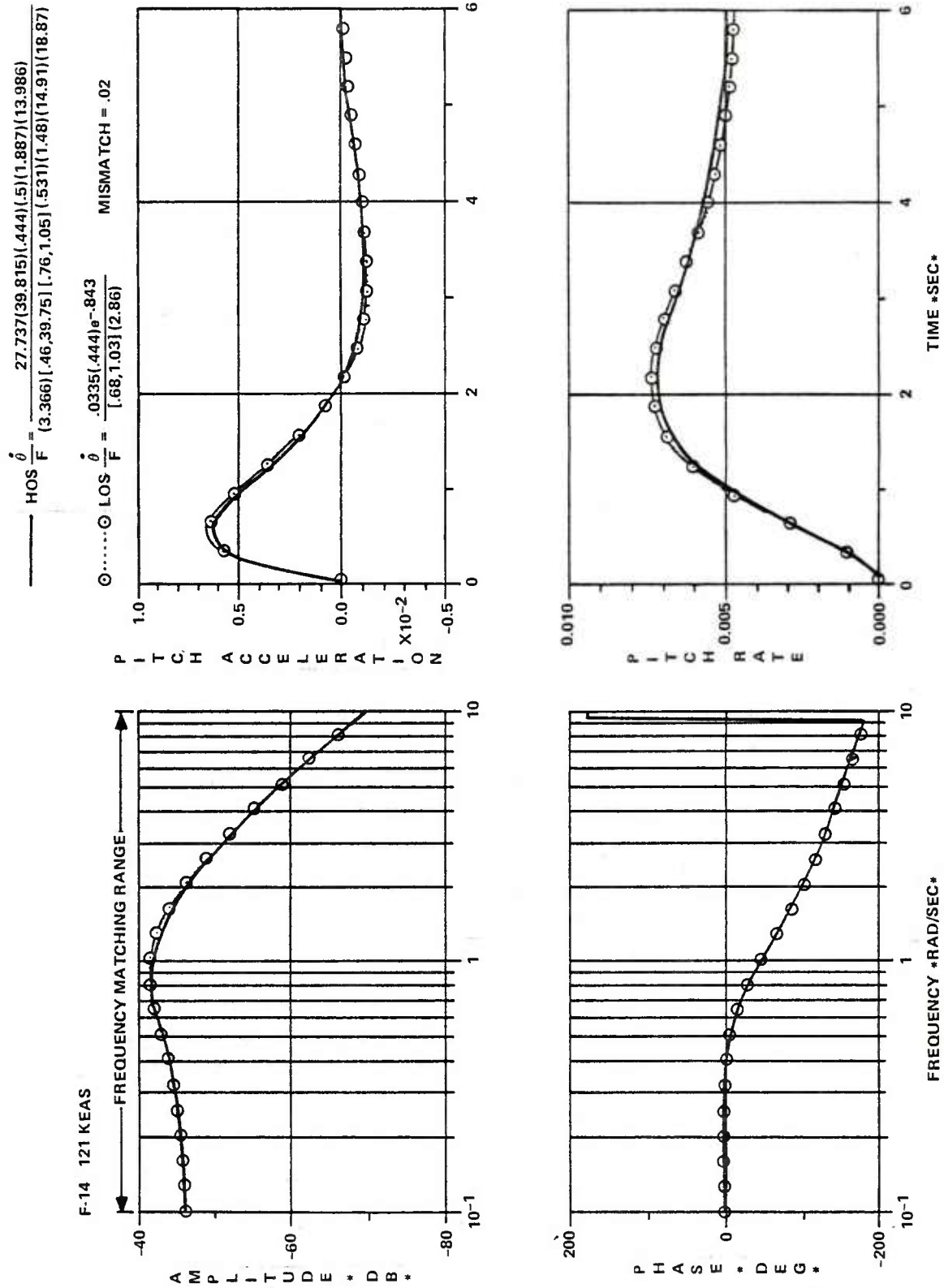


Figure B-15 — F-14 Airplane First Over Third Order Pitch Response to Control Force Inputs - Configuration PA(2) - L_α - Fixed

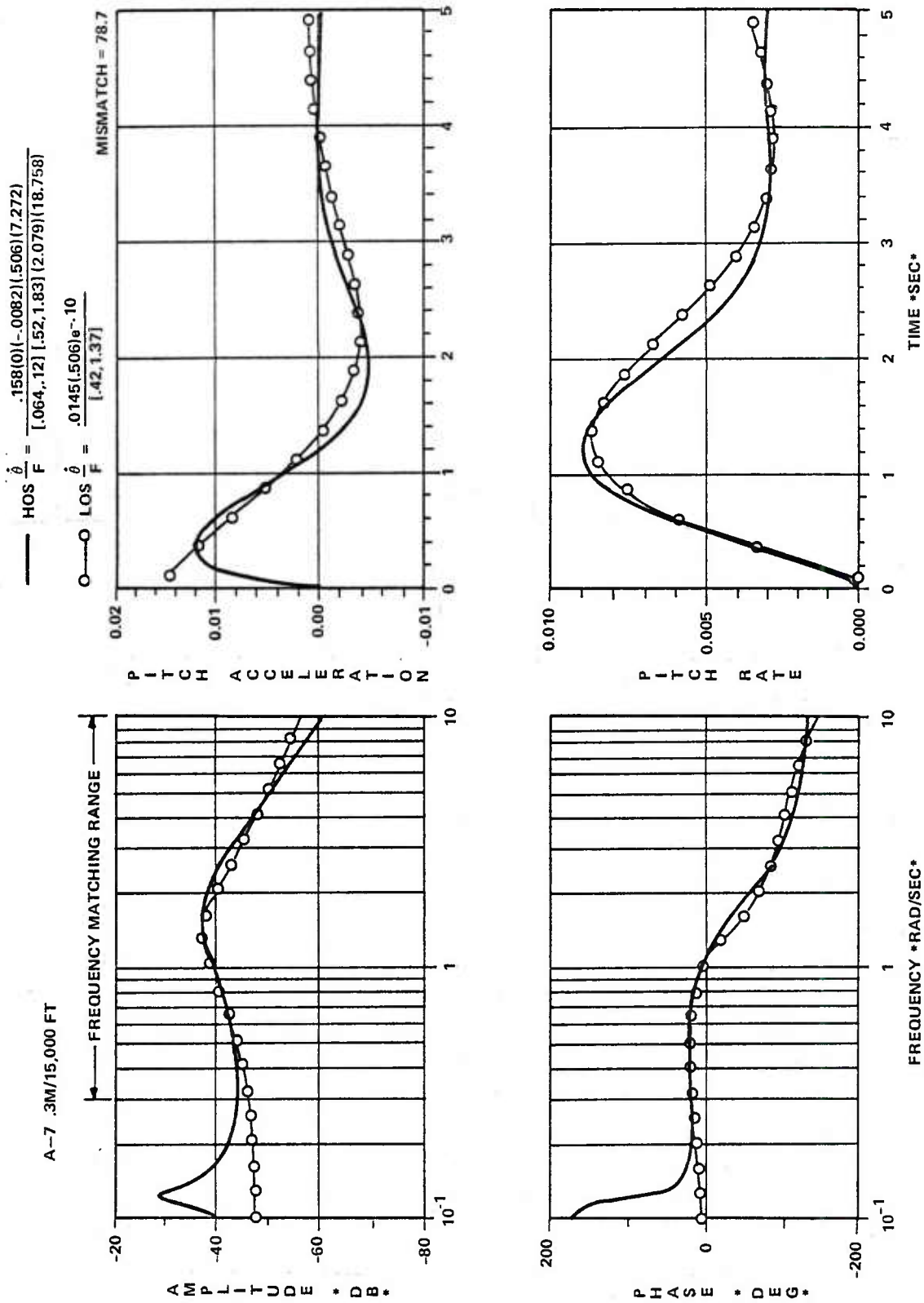


Figure B-16 — A-7 Airplane Pitch Response - L_{α} Fixed

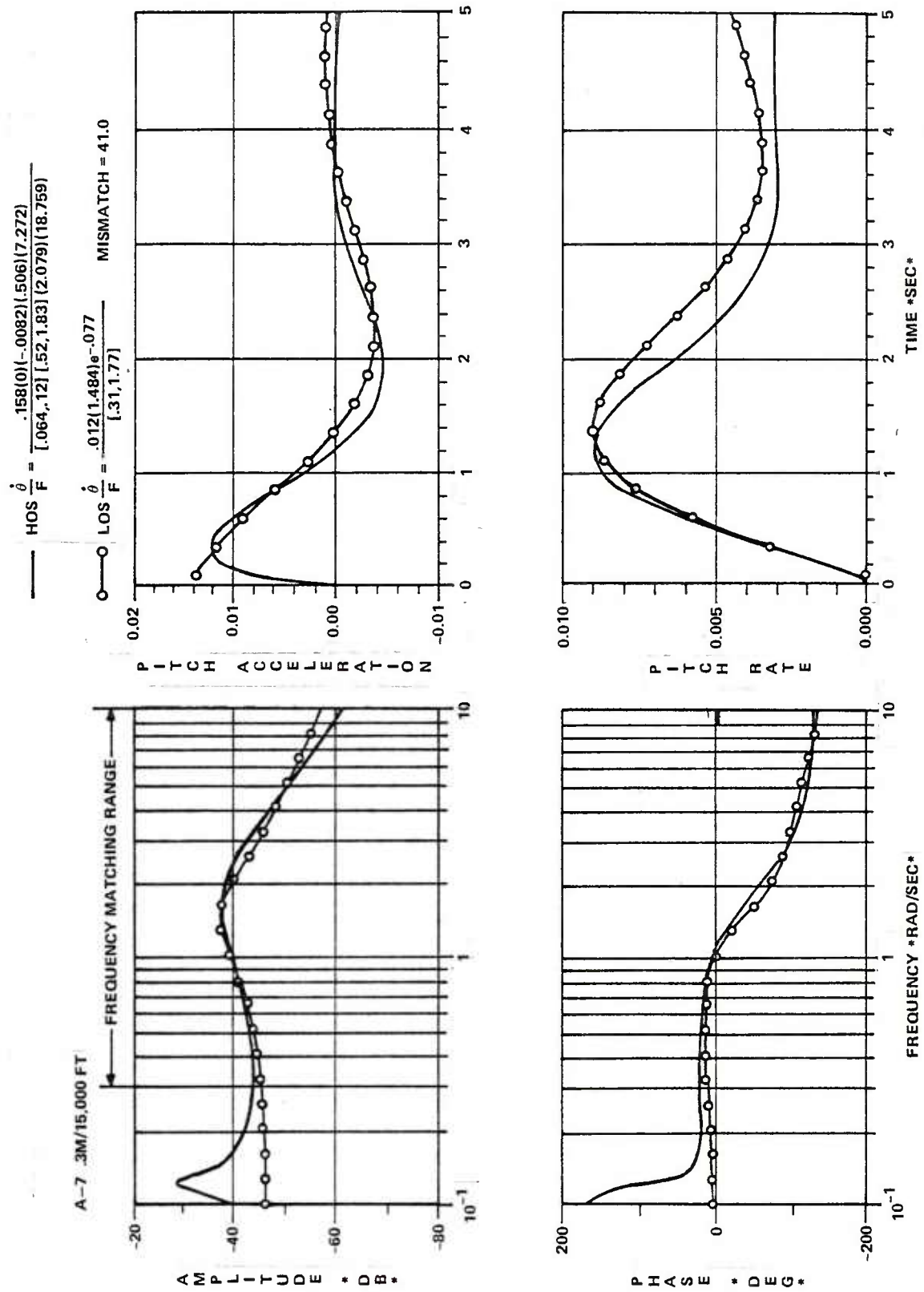


Figure B-17 — A-7 Airplane Pitch Response - L_{α} Free

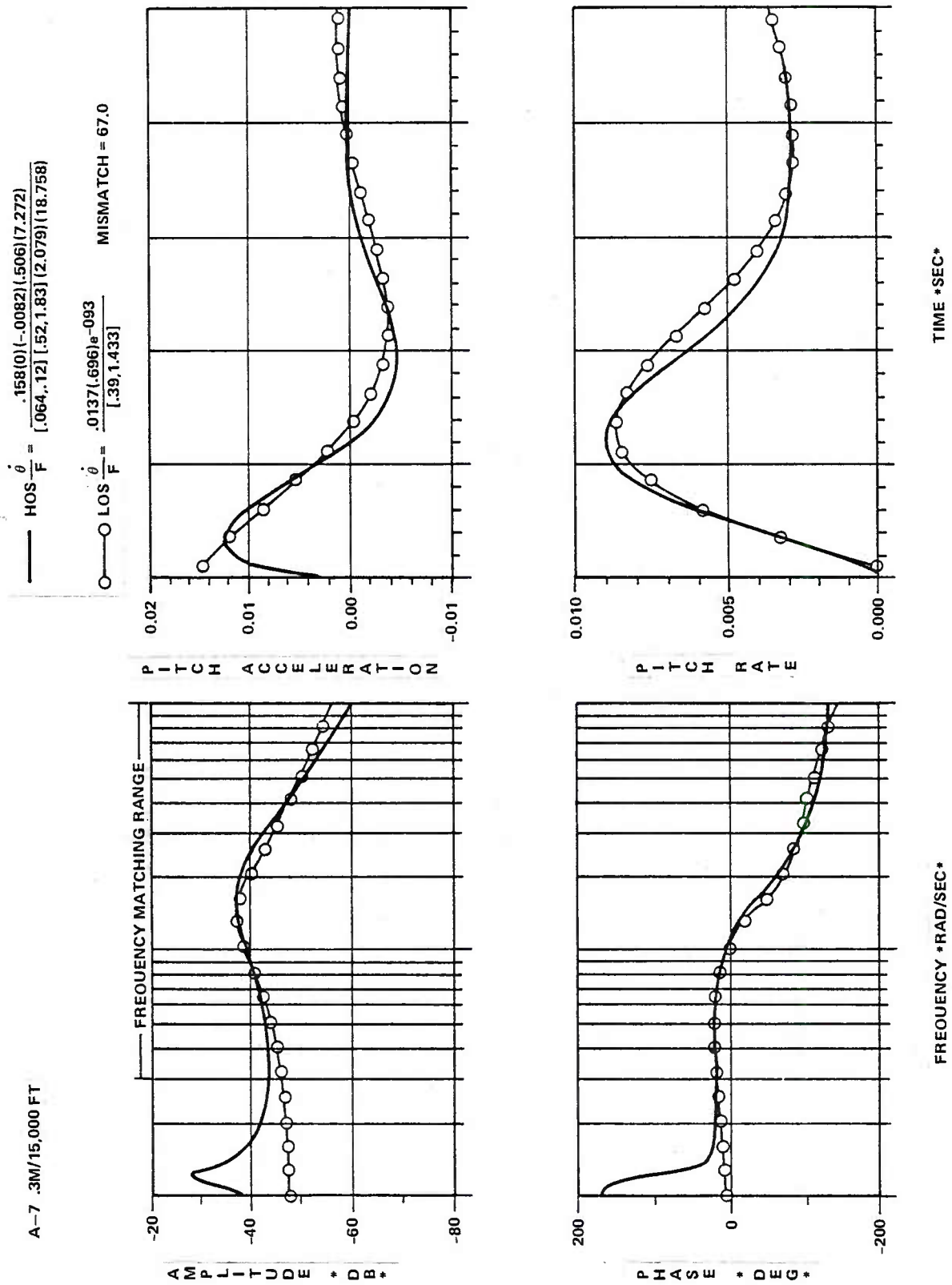


Figure B-18 — A-7 Airplane Pitch Response - Simultaneous $\dot{\theta}$ and Nz Matching

$$\text{HOS } \frac{NZ}{F} = \frac{.08(-.0042)(-.05)(340)(7.272)}{[.064, .12][.52, 1.83][2.079](18.758)}$$

$$\text{LOS } \frac{NZ}{F} = \frac{2.198e^{-.096}}{[.39, 1.43]} \quad \text{MISMATCH} = 90.3$$

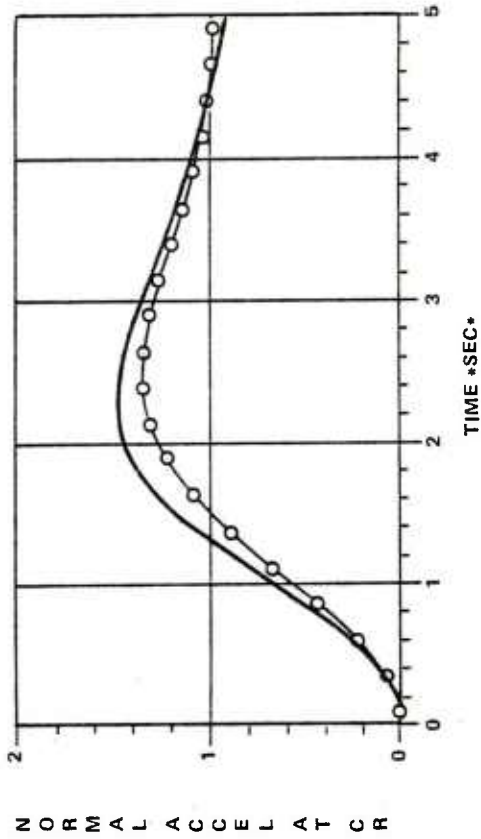
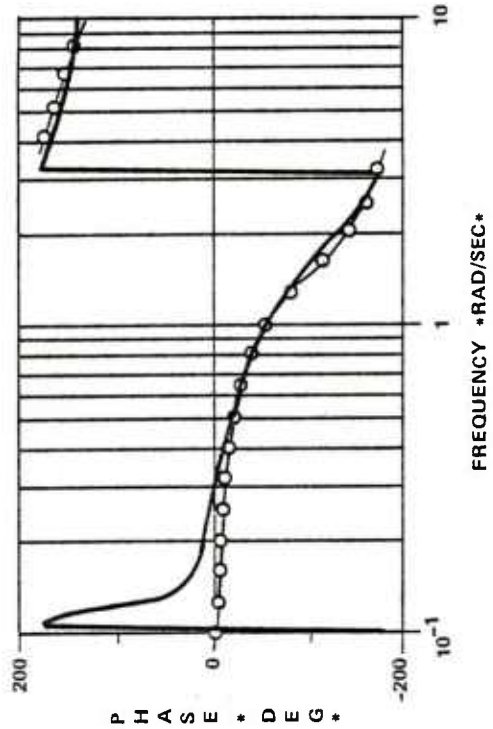
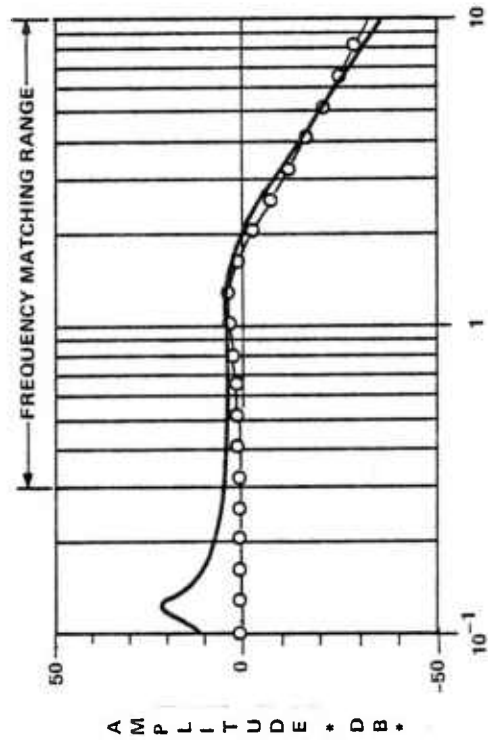


Figure B-19 — A-7 Airplane Normal Acceleration Response Simultaneous θ and Nz Matching

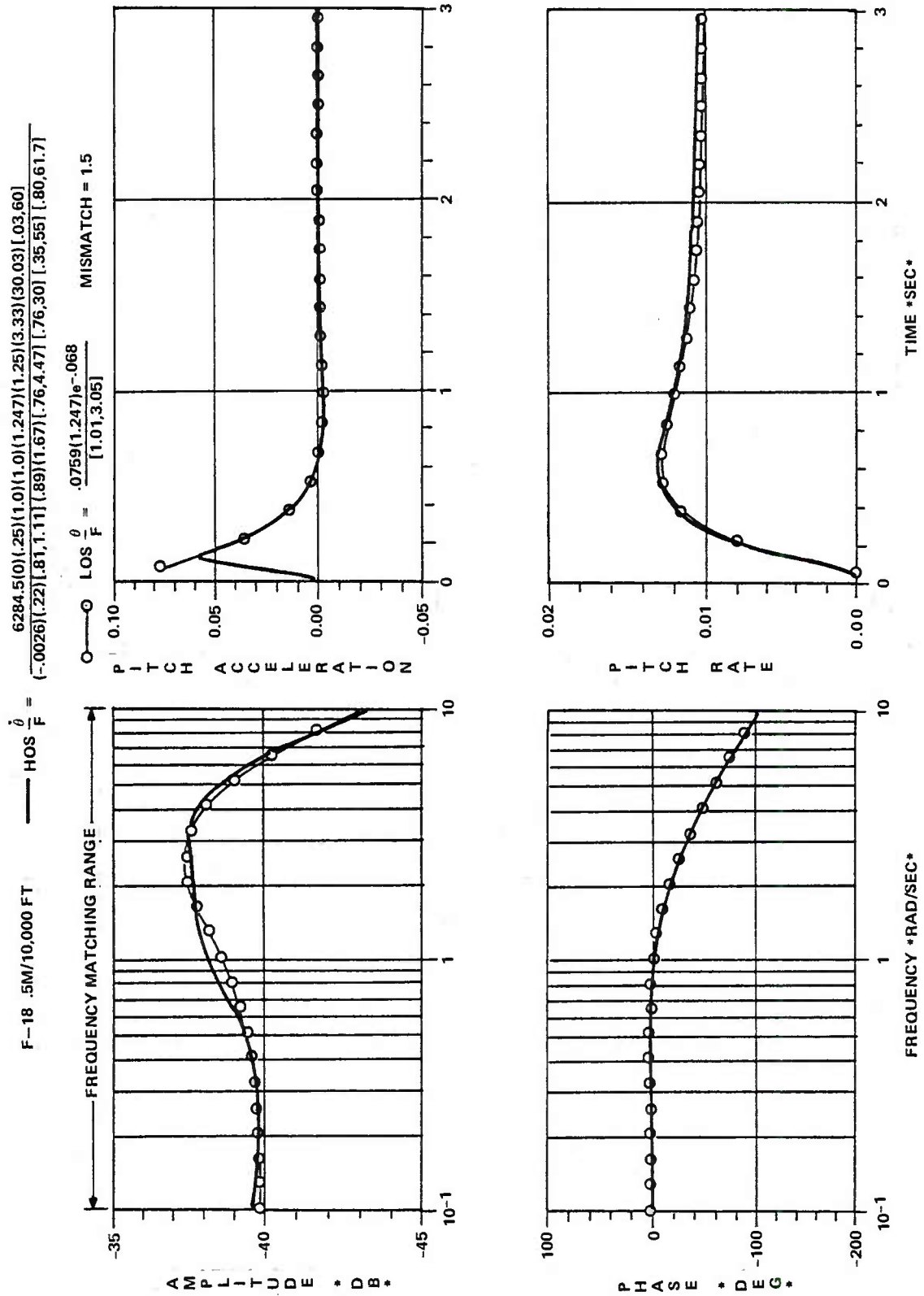


Figure B-20 — F-18 Airplane Pitch Response - Configuration CR - L_{α} Fixed

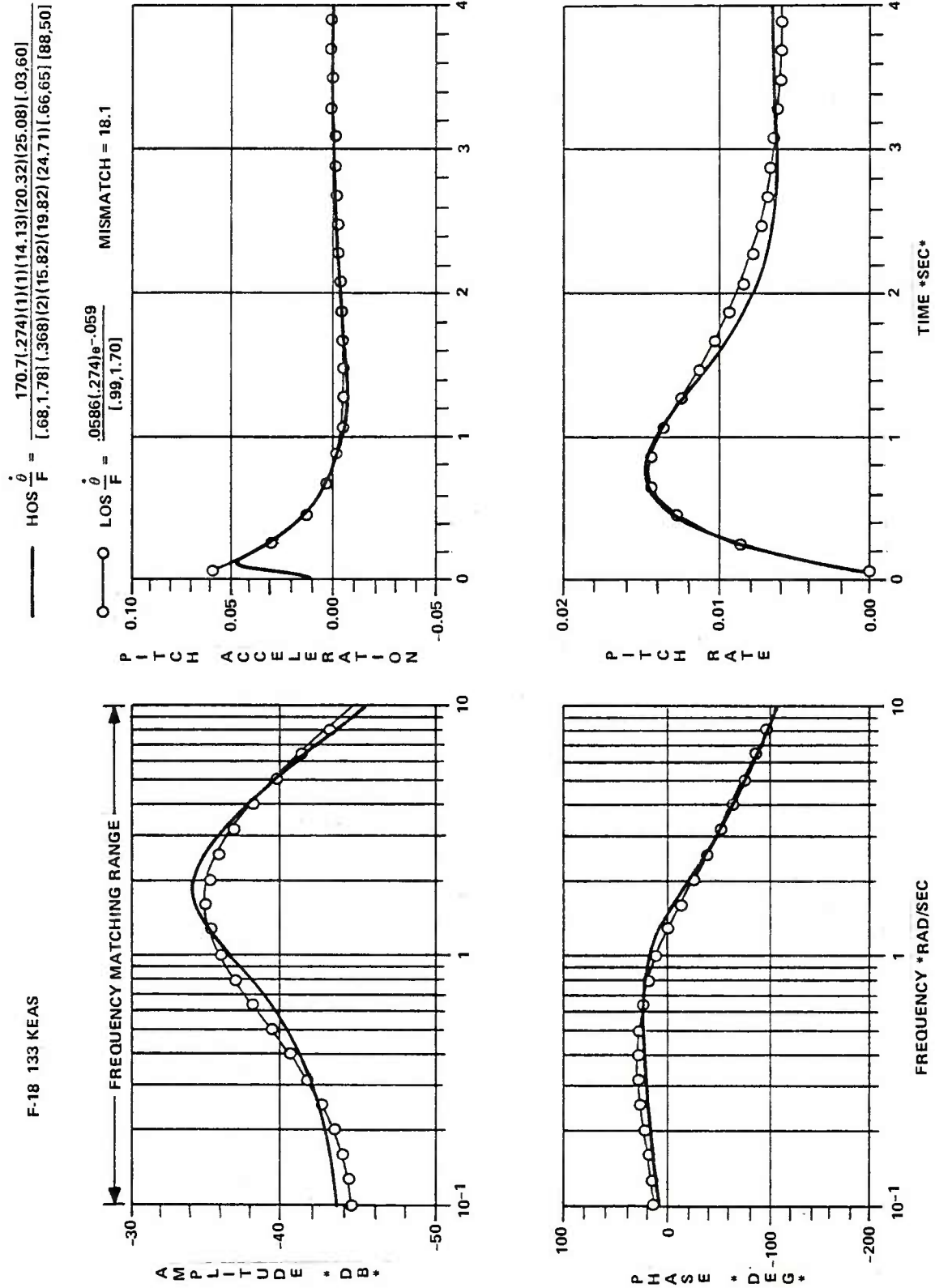


Figure B-21 — F-18 Airplane Pitch Response - Configuration PA - L_{α} Fixed

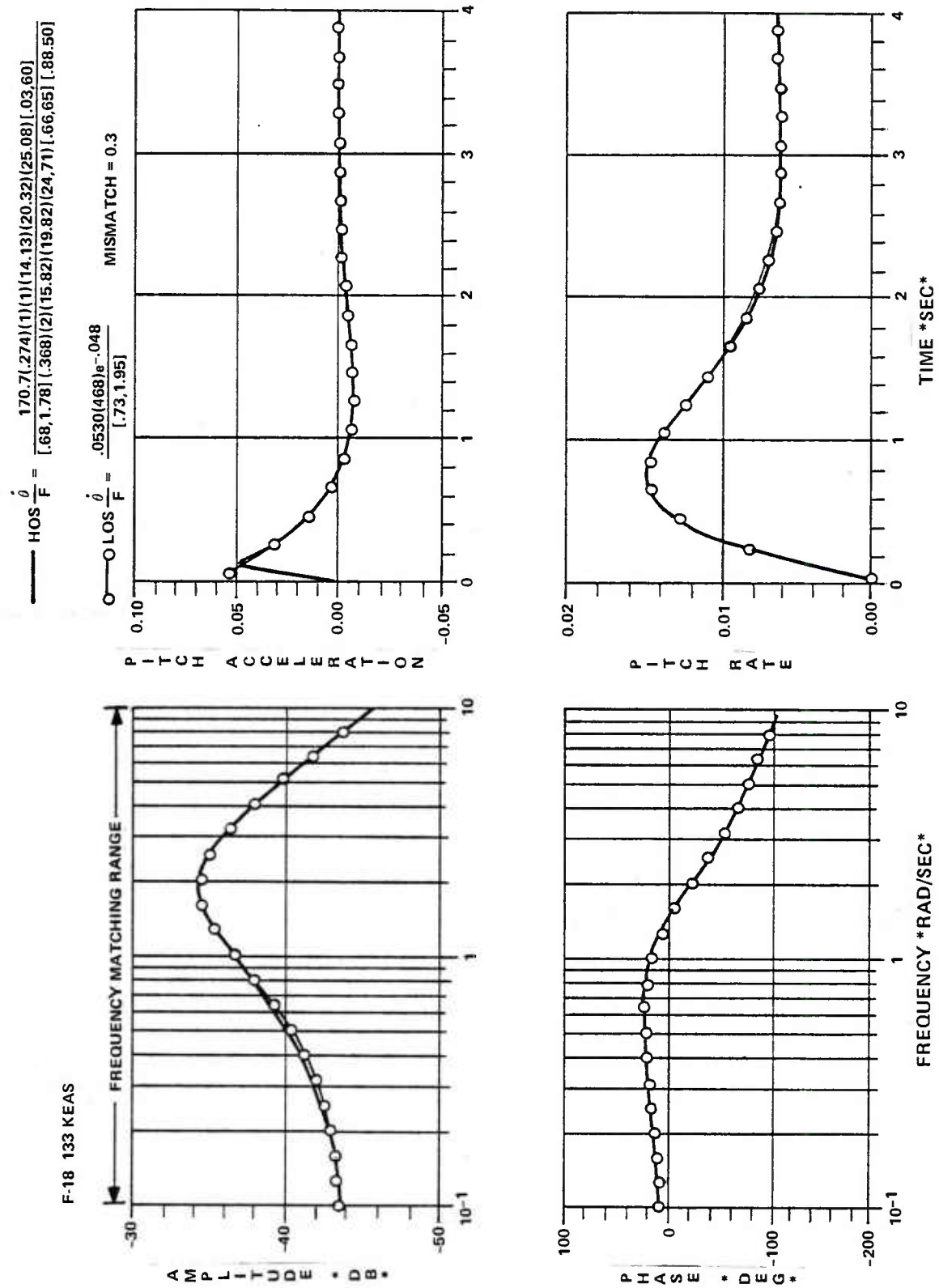


Figure B-22 — F-18 Airplane Pitch Response - Configuration PA - L_α Free

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$$\text{HOS } \frac{\dot{\theta}}{F} = \frac{170.7(.274)(1)(1)(14.13)(20.32)(25.08)[.0360]}{[.68, 1.78][.368](2)(15.82)(19.82)(24.71)[.66, 65][.88, 50]}$$

$$\text{LOS } \frac{\dot{\theta}}{F} = \frac{.0569(.347)e^{-.057}}{[.93, 1.79]}$$

MISMATCH = 8.3

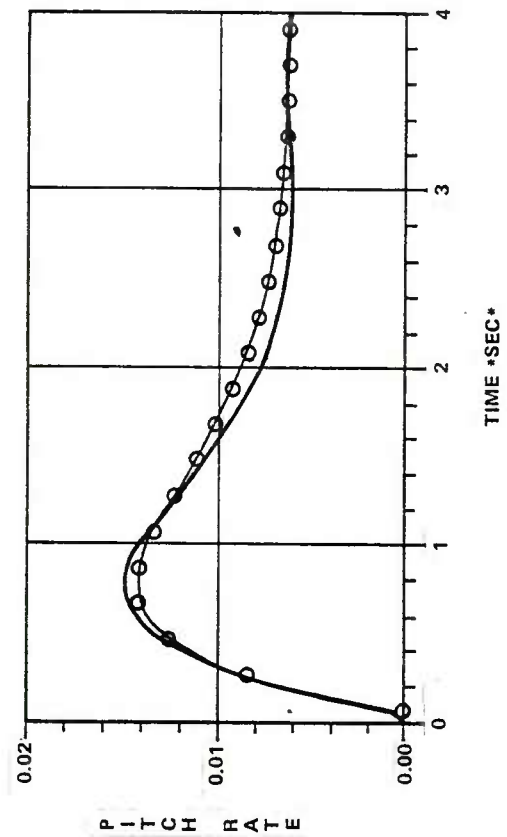
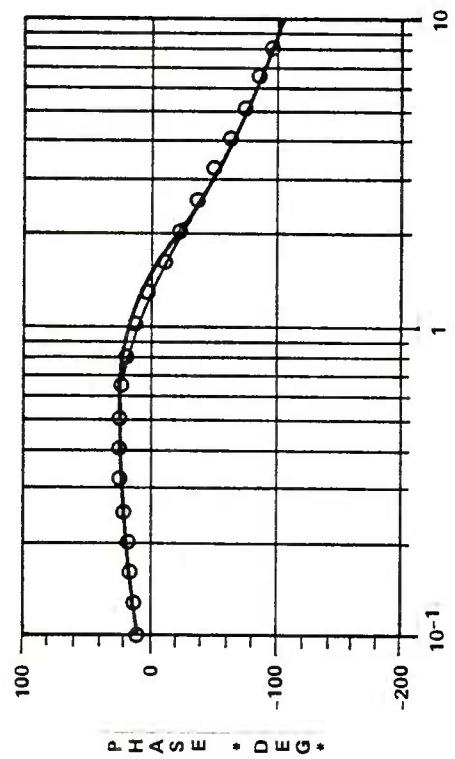
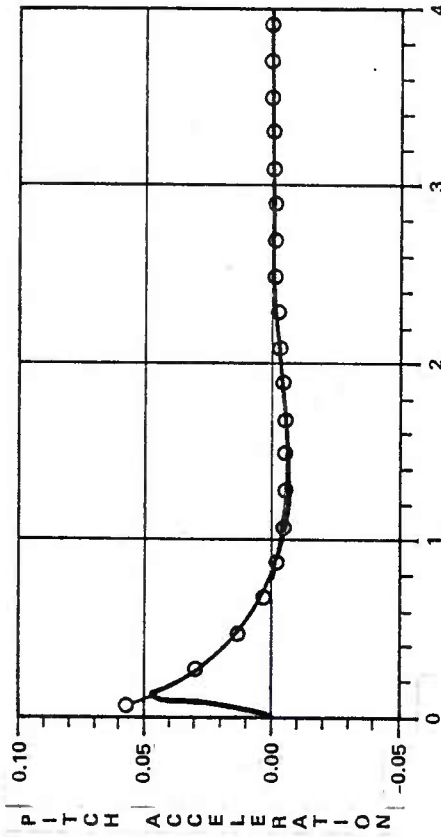
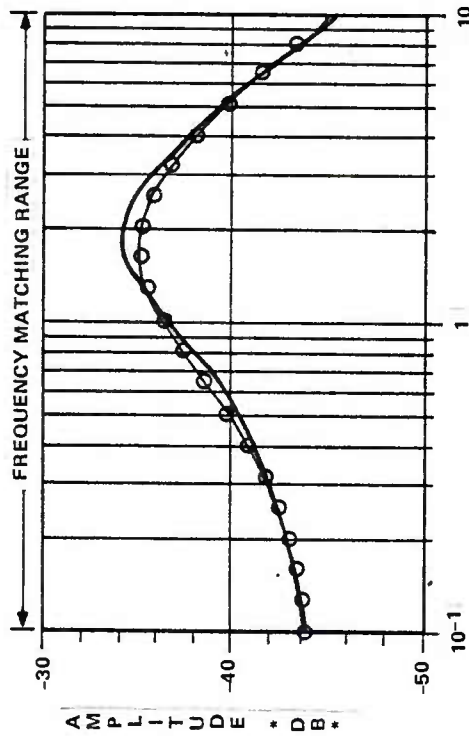


Figure B-23 — F-18 Airplane Pitch Response - Configuration PA - Simultaneous $\dot{\theta}$ and N_z Matching

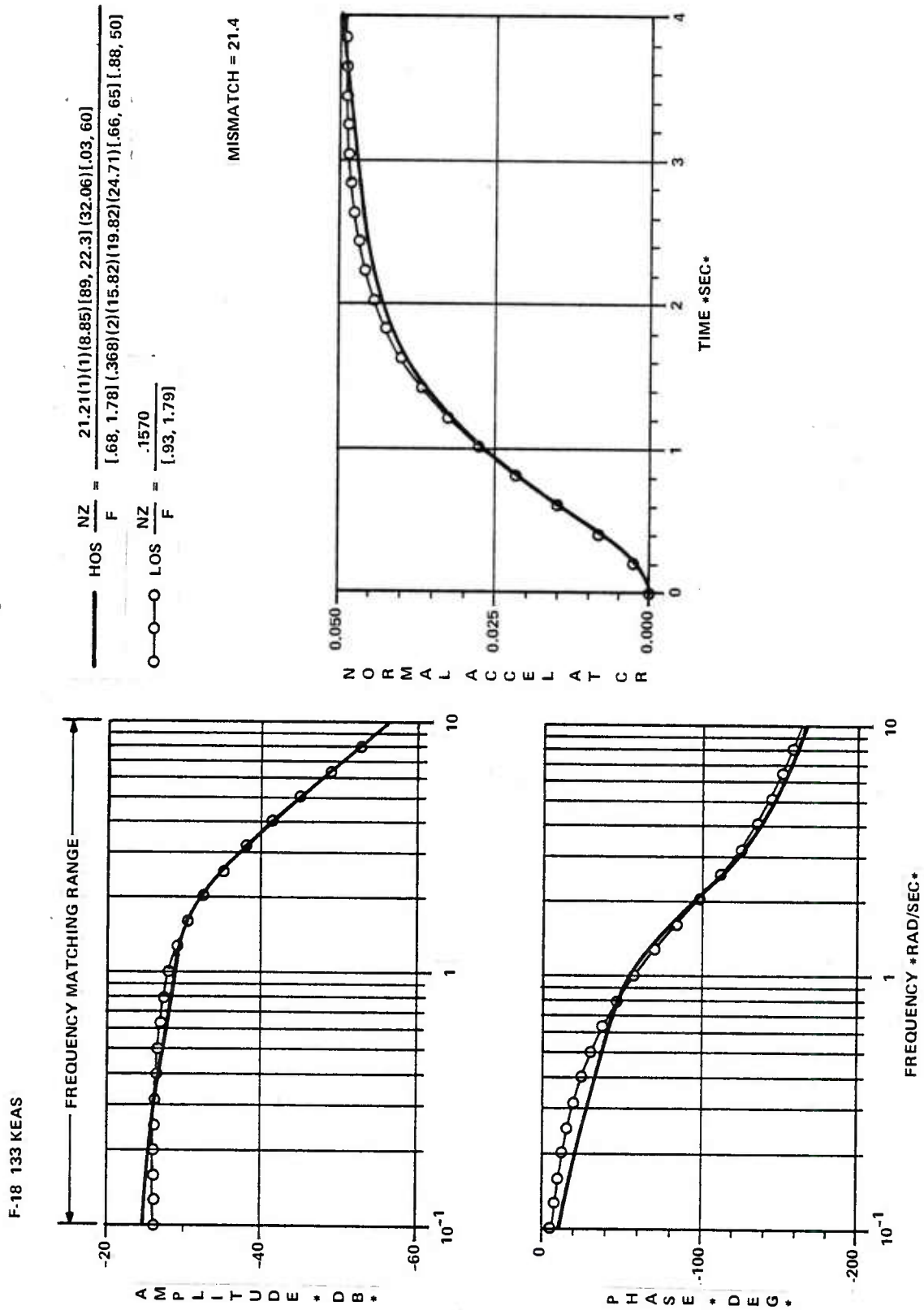


Figure B-24 — F-18 Airplane Normal Acceleration Response - Configuration PA - Simultaneous θ and Nz Matching

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